Solving the conflict of interests in the placement of transformers in the integrated distribution networks; a stochastic bi-level approach

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One of the important issues in power system planning is distribution network expansion planning (DNEP). This planning is always done in two steps. In the first step, planning options are suggested by simple mathematical models, and in the second step, these proposed models are subjected to a more detailed analysis, including stability issues. In this paper, a bi-level model is proposed for the DNEP, which can simultaneously consider both steps. At the lower-level, which is the low-voltage network, the expansion planning is done, and at the upper-level, which is the medium-voltage distribution network, the voltage stability index (VSI) is checked. The factor that causes conflict between the two levels is the size and location of distribution transformers so each level tends to determine the location and size of transformers according to its desire. To show the efficiency of the proposed model, in three different scenarios by considering the uncertainty in load demand, the problem is solved and the necessary comparisons are made in this regard.

keywords: Distribution network expansion planning, Distribution transformers placement, Tabu search algorithm, Uncertainty, Voltage stability index

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Nomenclature

Parameters

Ω_{DT}	Set of available transformers
Ω_{ES}	The set of lines in the low-voltage network
Ω_{is}	The points connected with the i-th point in the low-voltage network
Ω_{ND}	Set of new transformers
Ω_{NG}	Set of distributed generation units
Ω_{NS}	Set of medium-voltage lines
Ω_{PN}	Set of medium-voltage network points
Ω_{SC}	Existing and new lines in the low-voltage network
Ω_{SN}	Set of low-voltage network points
Ω_{TD}	Set of Transformer type
Ω_{TG}	Set of distributed generation units type
Ω_{TS}	Set of low-voltage lines type
$C^{ES}_{ij,c}$	The fixed cost of expansion existing lines between points i and j in the low-voltage network of type c
$C^{ND}_{i,d}$	Fixed cost of new transformer at point i of type d
$C^{NG}_{i,g}$	The fixed cost of the new distributed generation unit at point i of type g
$C^{NS}_{ij,c}$	The fixed cost of the new low-voltage line between points i and j of type c
$I^{\max}_{ij,c}$	Maximum flow between points i and j of low-voltage line type c
$I^{\max}_{ij,p}$	Maximum flow between points i and j of medium-voltage line type p
R^{p}_{ij}	Line resistance between points i and j of p-type medium-voltage line
R^{s}_{ij}	Line resistance between points i and j of s-type low-voltage line
$S^{cu}_{i,d}$	Copper losses of a transformer at point i of type d
$S^{fe}_{i,d}$	Iron losses of a transformer at point i of type d

$S_{i,d}^{\max}$	The maximum power of a type d transformer at point i
$S_{i,g}^{\max}$	The maximum power of a distributed generation unit of type g at point i
$S_{i,l}^{SD}$	Load power at point i for load level l
$V_{i}^{\max}$	Maximum voltage at point i
$V_{i}^{\min}$	Minimum voltage at point i
$V_{i-abcn}^{\max}$	Maximum voltage at point i for phases a, b and c
$V_{i-abcn}^{\min}$	Minimum voltage at point i for phases a, b and c

Variables

$\sigma_{ij,c}^{ES}$	Line expansion variable between points i and j of type c in low-voltage network
$\sigma_{i,d}^{ND}$	Variable installation of transformer at point i of type d
$\sigma_{i,g}^{NG}$	Variable installation of distributed generation at point i of type g
$\sigma_{ij,c}^{NS}$	Variable installation of the line between points i and j of type c in the low-voltage network
$I_{ij,l}$	Flow between two points i and j for load l in medium-voltage network
$I_{ij,l}^{abcn}$	Flow between two points i and j at load level l in low-voltage network for phases a, b and c
$S_{i,l}^{DT}$	Power injected into transformer point i for load level l
$S_{i,l}^{G}$	Injected power from distributed generation unit point i for load level l
$S_{i,l}^{S}$	Injected power from substation point i for load level l
$V_{i,l}$	Voltage of point i in medium-voltage network for load level l
$V_{i,l}^{abcn}$	Voltage of point i in the low-voltage network at the load level l in phases a, b and c

Abbreviation

LV	Low voltage
MV	Medium voltage
LL	Lower level
UL	Upper level
KKT	Karush Kuhn Tucker
TSA	Tabu search algorithm
MCS	Monte Carlo simulation

1. INTRODUCTION

A. Motivation and aim

Energy planning refers to a set of activities that are carried out on a macro level to study the interrelationship between the energy sector and other economic sectors with an emphasis on environmental considerations to create coordination between supply and demand. In every country, one of the important issues in government policy is energy planning and management. Large-scale energy planning depends on many factors, including economic development prospects, political approaches, macro-management issues, economic situation, etc. According to Fig. (1), one of the basic branches of energy planning is power system planning, which can be done even in a 50-year period. This planning includes generation expansion planning, transmission network expansion planning, and finally distribution network expansion planning (DNEP). The electricity industry is one of the most vital industries of a country. In the meantime, electrical energy distribution networks are the meeting point of the electricity industry subscribers, and the problems of the distribution system in this industry will be considered the problem of the entire electricity industry from the point of view of consumers.

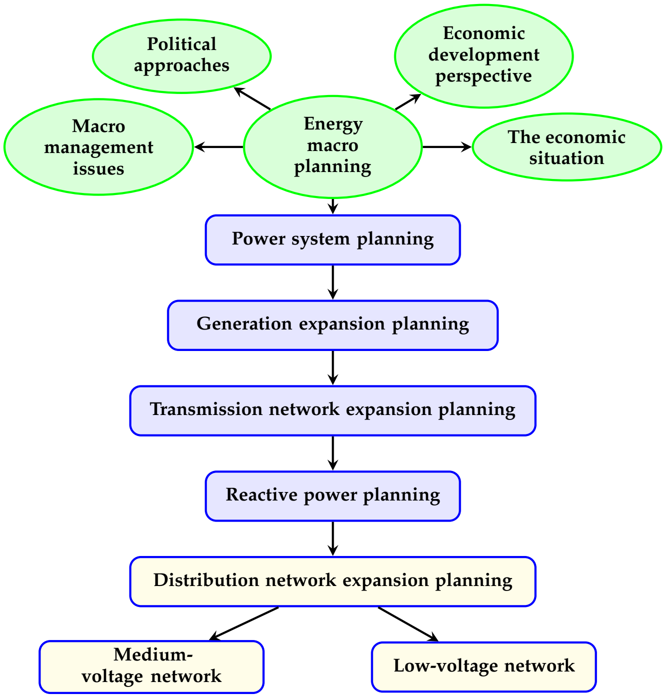


Fig. 1. Energy planning procedure

The increasing development, the lack of correct forecasting of this process, and the backwardness of technology have always brought problems in the electrical energy distribution system. Since the largest loss of electrical energy occurs in the distribution system and also due to the direct connection of this network with consumers, the management and operation of this network have a special place in power system planning. Also, lack of proper design, directing the system without planning, and setting goals without project control will result in damage to the national capital, energy losses, dissatisfaction, and pessimism of the subscribers. So, from this point of view, distribution network expansion planning is a special and important issue. This system is not only important in terms of the quantity of electrical energy distribution but also in terms of the provision and continuity of electricity supply with valid standards in the desired quality. An important point in distribution network planning is that in the first stage, planning is done by high-level information, and in the second stage, the proposed plans resulting from the first stage are evaluated more carefully, including operation and stability issues, and finally the desired plan is selected. The important point here is that due to the interruption of planning in two distinct stages, a plan that leads to lower investment and operating costs may not be optimized due to technical issues. In other words, in the second stage, a plan that leads to investment cost and optimal operation should not be chosen due to technical issues. Therefore, it is necessary to integrate these two planning stages in the distribution networks because they are not separate and the smallest change in each planning stage affects the other stage as well. In this paper, it has been tried to analyze a special type of planning in the distribution network by presenting a bi-level model.

The electrical energy distribution network is made up of two sections: medium voltage and low voltage. These two networks are connected through distribution transformers. Naturally, planning in these two networks is not independent from each other and the smallest change in each network affects the other network as well.

In this paper, a model is presented that takes into account network expansion planning, including feeder expansion and the allocation of distributed generations (DGs) at the low voltage level, and the effect of this simultaneous planning on the voltage stability of the medium-voltage network is investigated. In this case, both stages of distribution network expansion planning are done simultaneously. The important point in this issue is that what is the variable or the parameter connecting these two networks, or in other words, what is the communication bridge to check the optimal response in both networks? To answer this question, it should be said that communication bridges are distribution transformers that may have different capacities and locations from the point of view of each network. In other words, where is the optimal location and capacity of distribution transformers from the point of view of each network? Therefore, from this conflict, where is the optimal location and capacity of power transformers, the bi-level problem will be investigated. The main motivation of this paper came from the fact that in most of the previous studies, both medium and low voltage distribution networks were considered independent of each other and planning was done separately in them, while these two networks in planning discussions are not independent of each other at all.

In this paper, a bi-level model for the DNEP problem is proposed, which integrally considers both medium-voltage and low-voltage networks in the presence of electric load uncertainty. In the first level, the voltage stability criterion is optimized, and in the second level, the investment and operation costs are optimized. The factor that connects both the first and second levels is the location and size of distribution transformers. To check the uncertainty of electrical load, the Monte Carlo simulation (MCS) approach is used.

B. Literature review and contributions

In [1], a bi-level model is proposed for the DNEP problem, where the upper-level goal is to maximize the level of social welfare and the lower-level goal is to increase the participation of the owners of distributed generation resources in the production of electric power. In [2], a probabilistic bi-level model is proposed for the DNEP problem that quantifies and controls the level of economic risk associated with renewable energy sources. The bi-level model presented in [3] is in the presence of DGs and storages; so the upper level includes long-term investment decisions and the lower level includes short-term exploitation strategies. In [4], a bi-level model of distribution is presented, in which the upper-level distribution companies decide on their investment strategies, which leads to a demand signal for the lower-level sector. In [5], a bi-level model with several lower levels is presented, in which the upper level considers reliability, and other factors participating in the expansion are carried out at the lower level. In [6], a new bi-level model is proposed for the DNEP problem considering the different objectives of the distribution network operator and the electric vehicle parking lot owners. In [7], a bi-level model is presented to minimize both investment and operational costs in both the medium and low voltage networks. The upper level is dedicated to minimizing costs in the medium voltage network, while the lower level focuses on the same for the low voltage network, including considerations for pollution emissions. In [8], using the genetic algorithm the presented bi-level model to reduce investment costs in both levels is solved. In [9], the integrated DNEP problem in the presence of distributed generation resources, uncertainties, and reliability is considered. In [10, 11], the DNEP problem is solved using mixed integer linear programming and a heuristic approach, respectively. In [12, 13], an integrated programming problem is applied to improve reliability. In [14], the integrated DNEP problem in the presence of DGs is considered. A complete description of different methods and approaches in the field of DNEP issues is given in [15]. Ref. [16] focuses primarily on the implications of voltage limits and capital expenditures in distribution network planning. While it provides insights into financial considerations, it lacks a detailed exploration of the planning process itself. The absence of methodological clarity and scenario analysis limits its applicability in addressing the complexities involved in grid management. Ref. [17], adopts a hybrid approach utilizing a mixed-integer non-linear model focused on the optimal integration of battery energy storage systems and renewable energy sources within a three-phase distribution system. Its objective function encompasses the allocation and selection of various components, including substations, transformers, and branches, while also addressing operational aspects such as load balancing and the uncertainties associated with renewable energy sources, demand, and energy prices. In contrast, the second article presents a bi-level model for distribution network Expansion Planning , where the lower level concentrates on expansion planning for the low-voltage network, and the upper level assesses voltage stability in the medium-voltage network. The objective functions in this model are more focused on ensuring voltage stability and minimizing conflicts related to transformer sizing and placement.

The gaps of previous studied are as follows:

- Lack of Focus on Voltage Stability: While previous studies primarily focus on maximizing social welfare, managing economic risks, or reducing investment costs, none specifically target maximizing voltage stability in medium-voltage networks. This is a critical aspect, as voltage stability is essential for the reliable operation of power systems.

- Integration of Distribution Transformers: Most existing models do not consider the size and location of distribution transformers (DTs) as a connecting variable between the upper and lower levels of decision-making. Our approach uniquely incorporates DTs, which allows for a more integrated solution that addresses both operational costs and voltage stability.

- Holistic Approach to DNEP: The majority of the reviewed literature adopts either a single-objective or a multi-level approach without adequately integrating all relevant factors, such as reliability and economic considerations, in a cohesive manner. Our model presents a bi-level structure that simultaneously addresses these factors, providing a more comprehensive solution to the Distribution Network Expansion Planning (DNEP) problem.

- Addressing Uncertainties: While several references address uncertainties related to renewable energy sources and electric load, the proposed model takes a more holistic view by integrating these uncertainties specifically in the context of voltage stability and operational costs, which has not been sufficiently explored in prior research.

In this paper, the objective function of the first level is to maximize the voltage stability index in the medium-voltage network, and the objective function of the lower level is to minimize the cost of operation and investment of the low-voltage network. The variable that connects these two levels is the size and location of distribution transformers (DTs). In the next section, this structure is fully explained.

To solve bi-level problems, the Karush Kuhn Tucker (KKT) method or binary variables is used, but the radial condition of the distribution network makes the problem non-linear and cannot be solved by mathematical methods; However, all functions and constraints are linear. Therefore, according to the constraint of the radial distribution network, the problem will be solved by one of the meta-heuristic methods called the Tabu search algorithm (TSA), and an attempt will be made to bring the optimal solution as close as possible to the global optimum. The contributions of this paper can be listed as follows:

- Integrated planning of medium-voltage and low-voltage networks
- Using the voltage stability index (VSI) at the medium-voltage level
- Using the size and location of distribution transformers as a parameter connecting upper and lower levels
- Expansion of the distribution network at the low-voltage level, including the expansion and construction of feeders as well as DG sources
- Incorporating load uncertainty in the proposed bi-level model
- Using the TSA algorithm in the problem-solving

C. Paper organization

The organization of this paper is as follows: in the second 2, the structure of the proposed model is fully described, and in the section 3, the modeling of the proposed structure is presented. In the section (4), the method of solving the problem is given. In the section (5), the numerical results and validation of the proposed model are given, and finally, in the section (6), the results of this paper are given.

2. PROPOSED STRUCTURE PLANNING

A bi-level programming is a special type of hierarchical optimization problem in which the optimization problem of each level depends on the other level. This planning has two levels including an upper-level optimization problem and a lower-level optimization problem.

The main philosophy of this type of planning is due to the conflict that occurs between two levels, and this planning tries to resolve this conflict in a way that is acceptable to both levels.

In this paper, the existing conflict is where the distribution transformers should be placed from the point of view of both medium-pressure and low-voltage networks because due to the technical and operational issues of each of the medium-voltage and low-voltage networks, they tend to place the transformers in their desired location.

If the location of transformers is done only from the point of view of the medium-voltage network, the losses of the low-voltage network may increase and increase the cost of operation in the low voltage network, or if the location of the transformers is done from the point of view of the low-voltage network, inappropriate capacities for other elements medium-voltage network should be chosen which is not economical. Therefore, it is necessary to present a bi-level model for both medium-voltage and low-voltage networks so that it has the necessary compromise in both networks according to the criteria and gives an answer to the network planner that meets the limitations of both networks as much as possible.

It should be noted that in the applied bi-level model the medium-voltage network is considered an upper-level problem and the low-voltage network is considered a lower-level problem.

According to Fig. (2), in the applied bi-level model, the main movement is started by a forecast at the upper-level in such a way that the upper level first suggests a location and size for the distribution transformers ($S_{i,d}^{\max}$) and transfers these values to the lower-level. Next, the lower-level solves its optimization problem and obtains the power values injected into the transformers ($S_{i,l}^{\mathrm{DT}}$), and transfers these values to the upper-level.

Once the lower-level optimization problem is done, the low-voltage network returns the amount of power used for each transformer ($S_{i,l}^{DT}$) to the upper-level. At this stage, the medium-voltage network can be solved.

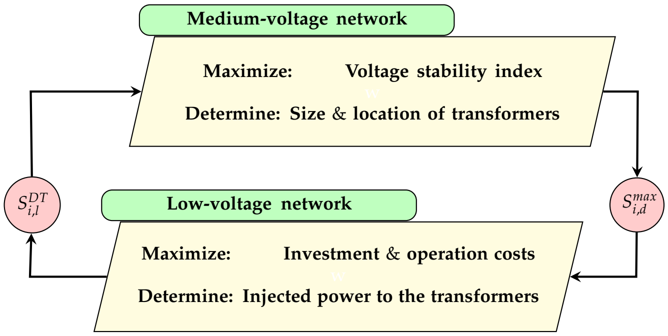


Fig. 2. The applied bi-level structure

3. MODELING

A. Voltage stability index

The voltage stability index (VSI) can be derived from a simple load flow in the radial distribution networks expressed in detail in [18, 19]. From Fig.(3), the following equation can be written:

$$I_{ij} = \frac{V_i - V_{i+1}}{R_i + jX_i} \tag{1}$$

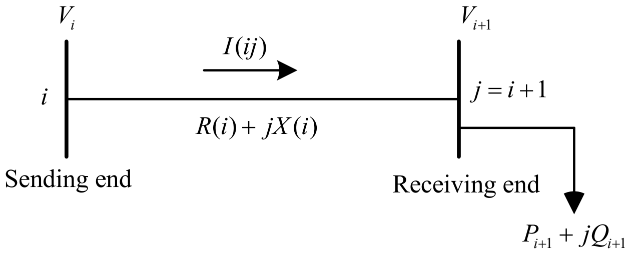


Fig. 3. Single line equivalent circuit of a distribution network

$$P_{i+1} - jQ_{i+1} = V^{*}_{i+1} I_{ij} \tag{2}$$

Therefore, it can be said:

$$\begin{aligned} &|V_{i+1}|^4 - \left\{|V_i|^2 - 2P_{i+1}R_i - 2Q_{i+1}X_i\right\}|V_{i+1}|^2 \\ &+ \left\{P^2{}_{i+1} + Q^2{}_{i+1}\right\}\left\{R_i^2 + X_i^2\right\} = 0 \end{aligned} \tag{3}$$

Suppose:

$$\mathbf{B} = |V_i|^2 - 2P_{i+1}R_i - 2Q_{i+1}X_i \tag{4}$$

$$\mathbf{C} = \left\{P^2{}_{i+1} + Q^2{}_{i+1}\right\}\left\{R^2{}_i + X^2{}_i\right\} \tag{5}$$

Now, the Eq. (3) can be expressed as follows:

$$|V_{i+1}|^4 - \mathbf{B}|V_{i+1}|^2 + \mathbf{C} = 0 \tag{6}$$

Solving the above equation leads to four answers as follows:

1. $0.707\left[B - \left\{B^2 - 4C\right\}^{1/2}\right]^{1/2}$
2. $-0.707\left[B - \left\{B^2 - 4C\right\}^{1/2}\right]^{1/2}$
3. $-0.707\left[B + \left\{B^2 - 4C\right\}^{1/2}\right]^{1/2}$
4. $0.707\left[B + \left\{B^2 - 4C\right\}^{1/2}\right]^{1/2}$

If the data values are in per unit (p.u) form, then the **B** term is always positive because the $2P_{i+1}R_i + Q_{i+1}X_i$ term is very small compared to the $|V_i|^2$ term. Also, the term 4**C** is very small compared to $\mathbf{B}^2$. Therefore, $\{\mathbf{B}^2 - 4\mathbf{C}\}^{1/2}$ is almost equal to **B**, and therefore two of the resulting answers (the first and second answers) for $|V_{i+1}|$ are almost equal to zero and are unacceptable answers. The third answer is also unacceptable because it is negative, and only the fourth answer, which is as follows, is an acceptable and positive answer:

$$|V_{i+1}| = 0.707\left[B + \left\{B^2 - 4C\right\}^{1/2}\right]^{1/2} \tag{7}$$

$$\mathbf{B}^2 - 4\mathbf{C} \geq 0 \tag{8}$$

Now, according to Eqs. (4), (5), and (8), it can be written that:

$$\begin{aligned} VSI &= |V_i|^4 - 4\{P_{i+1}X_i - Q_{i+1}R_i\}^2 \\ &\quad - 4\{P_{i+1}R_i + Q_{i+1}X_i\}|V_i|^2 \geq 0 \end{aligned} \tag{9}$$

In Fig. (4), the graph of the VSI in the distribution network for different values of the feeder parameters is shown.

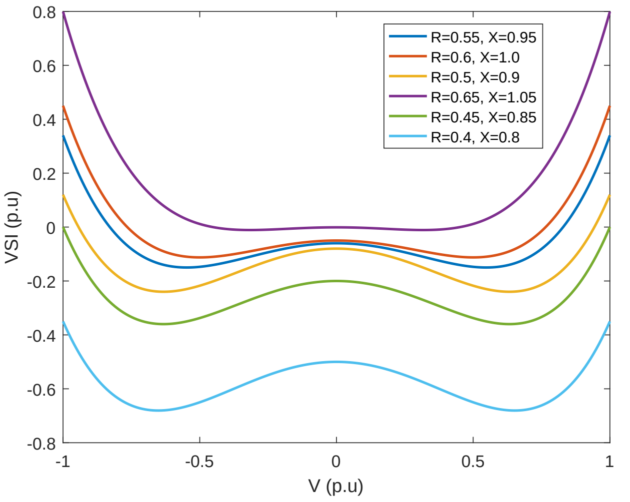


Fig. 4. The VSI in the distribution network for different values of the feeder parameters

B. Proposed optimization framework

The mathematical form of the proposed bi-level problem is presented in Eqs. (10)-(24), where the medium-voltage and low-voltage networks are represented by single-phase and three-phase models, respectively.

$$\max \quad VSI(i+1) = \sum_{i=1}^{NB-1}\begin{pmatrix} V_i^4 - 4(P_{i+1}X_i - Q_{i+1}R_i)^2 \\ -4(P_{i+1}R_i + Q_{i+1}X_i)^2 V_i^2 \end{pmatrix} \tag{10}$$

subject to:

$$S^S_{i,l} = S^{DT}_{i,l} + V_{i,l} I^*_{ij,l} \qquad \forall\, i \in \Omega_{PN}\,;\, \forall\, l \in \Omega_{NL} \tag{11}$$

$$V_i^{\min} \leq V_{i,l} \leq V_i^{\max} \qquad \forall\, i \in \Omega_{PN}\,;\, \forall\, l \in \Omega_{NL} \tag{12}$$

$$I_{ij,l} \leq I_{ij}^{\max} \qquad \forall\, ij \in \Omega_{PF}\;,\;\; \forall\, l \in \Omega_{NL} \tag{13}$$

$$\sum_{d \in \Omega_{TD}} \sigma^{ND}_{i,d} \leq 1 \qquad \forall\, i \in\, \Omega_{DT} \tag{14}$$

$$\min \begin{pmatrix} \displaystyle\sum_{ij\in\Omega_{NS}}\sum_{c\in\Omega_{TS}} C^{NS}_{ij,c}\sigma^{NS}_{ij,c} + \\ \displaystyle\sum_{ij\in\Omega_{ES}}\sum_{c\in\Omega_{TS}} C^{ES}_{ij,c}\sigma^{ES}_{ij,c} + \\ \displaystyle\sum_{i\in\Omega_{NG}}\sum_{g\in\Omega_{TG}} C^{NG}_{ig}\sigma^{NG}_{i,g} + \\ \displaystyle\sum_{l=1}^{nL}\sum_{ij\in\Omega_{SC}}\sum_{c\in\Omega_{TS}} R^c_{ij} \times \left[\left|I^{abcn}_{ij,l}\right|^2(\sigma^{NS}_{ij,c} + \sigma^{ES}_{ij,c})\right] + \\ \displaystyle\sum_{l=1}^{nL}\sum_{i\in\Omega_{DT}}\sum_{d\in\Omega_{TD}} \left[S^{fe}_{i,d} + S^{cu}_{i,d}\left(\frac{S^{DT}_{i,l}}{S^{\max}_{i,d}}\right)^2\right] \end{pmatrix} \tag{15}$$

subject to:

$$S^{DT}_{i,l} = S^{SD}_{i,l} - S^{G}_{i,l} + \sum_{j\in\Omega_{is}} V^{abcn}_{i,l}\left[\sum_{c\in\Omega_{TS}}\left(\sigma^{NS}_{ij,c} + \sigma^{ES}_{ij,c}\right) I^{abcn*}_{ij,l}\right] \tag{16}$$

$\forall\, i \in \Omega_{SN}\,;\, \forall\, l \in \Omega_{NL}$

$$I^{abcn}_{ij,l} \leq I^{\max}_{ij,c} \qquad \forall\, ij \in \Omega_{SC}\ ;\ \forall\, l \in \Omega_{NL}\ ;\ \forall\, c \in \Omega_{TS} \tag{17}$$

$$S^{DT}_{i,l} \leq S^{\max}_{i,d} \qquad \forall\, i \in \Omega_{DT}\ ; \forall\, l \in \Omega_{NL}\ ;\ \forall\, d \in \Omega_{TD} \tag{18}$$

$$\begin{aligned} S^{DT}_{i,l} = {} & \left[S^{fe}_{i,d} + S^{cu}_{i,d} \left(\frac{S^{DT}_{i,l}}{S^{\max}_{i,d}} \right)^2 \right] \\ & + \sum_{l=1}^{nL} \sum_{ij \in \Omega_{SC}} \sum_{c \in \Omega_{TS}} R^{c}_{ij} \left[\left| I^{abcn}_{ij,l} \right|^2 \right] + S^{SD}_{i,l} \\ & \forall\, ij \in \Omega_{SC}\ ; \forall\, l \in \Omega_{NL}\ ;\ \forall\, i \in \Omega_{DT} \end{aligned} \tag{19}$$

$$S^{G}_{i,l} \leq S^{\max}_{i,g} \qquad \forall\, i \in \Omega_{NG}\ ; \forall\, l \in \Omega_{NL}\ ;\ \forall\, g \in \Omega_{TG} \tag{20}$$

$$V^{\min}_{i-abcn} \leq V^{abcn}_{i,l} \leq V^{\max}_{i-abcn} \qquad \forall\, i \in \Omega_{SN}\ ; \forall\, l \in \Omega_{NL} \tag{21}$$

$$\sum_{c \in \Omega_{TS}} (\sigma^{NS}_{ij,c} + \sigma^{ES}_{ij,c}) \leq 1 \qquad \forall\, ij \in\ \Omega_{SC} \tag{22}$$

$$\sum_{g \in \Omega_{TG}} \sigma^{NG}_{i,g} \leq 1 \qquad \forall\, i \in\ \Omega_{NG} \tag{23}$$

$$\text{Radial distribution network} = 1 \tag{24}$$

The Eqs. (10)-(14) represent the upper-level problem or medium-voltage network problem, whose objective function according to Eq. (10) is to maximize the voltage stability index. The constraints of the upper-level problem are stated in Eqs. (11)-(14) that Eq. (11) represents the power balance constraint, Eq. (12) represents the bus voltage limitation, Eq. (13) represents the thermal capacity constraint of lines and Eq. (14) states that only one type of transformer can be installed in the desired bus. Eq. (15) is the lower-level objective function comprising five parts. The first part shows the cost of installing new lines, the second part shows the cost of upgrading existing lines, the third part shows the cost of installing distributed generations, the fourth and fifth parts shows the cost of losses including lines and transformers in this network, respectively. The lower-level constraints are given in Eqs. (16)-(24). Eq. (16) expresses the load balance constraint, Eq. (17) expresses the thermal capacity limitation of low-voltage network lines, Eq. (18) expresses the capacity limitation of distribution transformers, Eq. (19) expresses the amount of power injected into distribution transformers, Eq. (20) expresses the limitation of capacity of the installed distributed generations, Eq. (21) represents the voltage limitation of the buses, Eq. (22) represents that only one type of line can be installed between two buses, Eq. (23) represents that only one type of distributed generation can be installed in the bus, and Eq. (24) states that the network maintains its radial property after expansion.

4. SOLVING METHOD DESCRIPTION

The proposed optimization model is a mixed integer nonlinear programming and the Tabu search algorithm (TSA) is used to solve it. This algorithm starts from an initial answer. Then the algorithm selects the best neighboring solution among the neighbors of the current solution. If this answer is not in the forbidden list, the algorithm moves to the neighboring answer; otherwise, the algorithm will check a criterion called the breathing criterion. Based on the breathing criterion, if the neighbor's solution is better than the best solution found so far, the algorithm will move to it, even if that solution is in the forbidden list. After the algorithm moves to the neighbor answer, the forbidden list is updated; this means that the previous move by which the neighboring answer was moved is placed in the forbidden list to prevent the algorithm from returning to that answer and creating a cycle. The forbidden list is a tool in the TSA that prevents the algorithm from reaching the local optimum. After placing the previous move on the banned list, some previously banned moves will be removed from the list. Moving from the current solution to the neighboring solution continues until the termination condition (for example, the limit of the number of moves to the neighboring solution) is met. The flowchart of this algorithm is shown in Fig. (5).

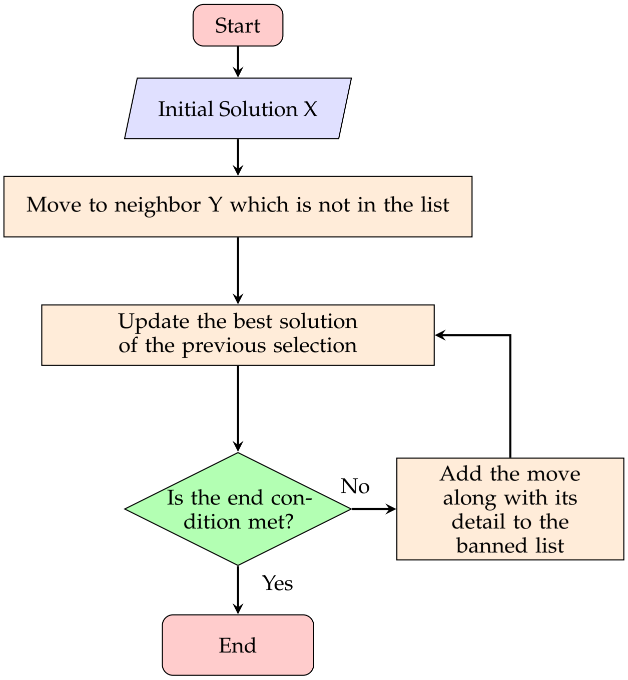
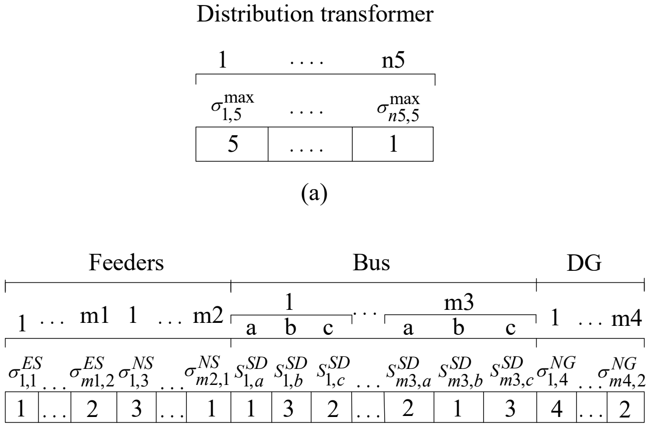


Fig. 5. The flowchart of the TSA

To properly code the medium-voltage and low-voltage levels, a new coding scheme is proposed according to Fig. (6). This vector in medium-voltage level consists of a part that is the location and capacity of the transformers. Similarly, the low-voltage level is coded applying a vector which contains the data on the low-voltage level.

The vector of the low-voltage section consists of three sections, the first section is the location and size of existing and new lines ($m_1 + m_2$ sizing), the second section is the loading location in every phase (m_3 sizing) and the third section composes the capacity and location of distributed generations (m_4 sizing). Integer numbers are specified in Fig. (6), which means that the



(a)

(b)

Fig. 6. Coding: (a): medium-voltage level, (b): low-voltage level

element is connected to the network in the number shown in each phase. The symbols a, b, and c represent the desired phases, for example, the number zero means that the element is not connected to the system, or the number one or two indicates that one or two of those elements are connected to the corresponding phase.

Electrical loads in distribution networks are associated with uncertainty due to uncertainty in their forecasting. Therefore, considering them makes the presented plan respond to the needs of the network more favorably during the planning period. To include the uncertain effect in the proposed problem, the scenario generation technique based on the Monte Carlo simulation method is used, and a detailed description of this technique is given in [20]. The initial configuration for both levels is obtained by starting from the transformer and at each stage a new branch is connected to the low voltage network. When a branch is connected, exploitation constraints are evaluated. This strategy stops when all buses are connected to the system.

It is emphasized that only feasible topologies are allowed in the initial configuration. The neighborhood structure is the upper level of bus voltage. For the lower level, neighborhood criteria are upgrade and installation of new lines, phase balance and installation of distributed generation sources. Transformers are not applied as adjacency criteria because they are links between any two systems in the proposed model.

To solve constrained optimization problems, there are different methods, one of which is resorting to the penalty function. In this method, it is allowed to violate the constraints of the problem to the suggested answers; But each answer has to pay a fine depending on the amount of violation. This penalty is implemented to worsen the response quality by manipulating the value of the objective function. For example, in minimization problems, the penalty function increases the value of the objective function and worsens the response. There are several ways to define the penalty function, but a series of general principles should be considered in designing this mechanism.

In this paper, the fitness functions are according to Eqs. (25) and (26). These relations are the sum of the objective functions and the penalty functions (penalty costs). In these relations, $f_P V$, $f_P I$, and $f_P S$ are respectively related to the crimes of violation of voltage, violation of thermal capacity, and violation of the capacity of transformers.

$$F_{fitMV} = \text{Eq.}(10) + \underbrace{f_P V_{MV}(\Delta_{V_{MV}}) + f_P I_{MV}(\Delta_{I_{MV}}) + f_P S_{MV}(\Delta_{S_{MV}})}_{\text{Penalty cost for upper-level}} \tag{25}$$

$$F_{fitLV} = \text{Eq.}(15) + \underbrace{f_P V_{LV}(\Delta_{V_{LV}}) + f_P I_{LV}(\Delta_{I_{LV}}) + f_P S_{LV}(\Delta_{S_{LV}})}_{\text{Penalty cost for lower-level}} \tag{26}$$

As mentioned in the second (2), first the initial configurations ($S^{\max}_{i,d}$) are proposed by the upper-level and these configurations are transferred to the lower-level. In the following, the lower-level solves its problem and evaluates its fitness function, in this way, the optimal configuration and the amount of power injected into the distribution transformers are determined ($S^{DT}_{i,l}$). Now, this lower-level response is transferred to the upper-level, and the upper-level also solves its problem and evaluates its fitness function. If the upper-level can achieve an optimal solution in its problem, then the stopping criterion is met and the complete solution of the problem is finished. The flowchart of the proposed problem solving method is shown in Fig. (7).

A. Scenario-based modeling of electrical load

In this paper, the stochastic programming method has been used to include the uncertainty in the electric load demand in such a way that first a set of scenarios is defined for the parameter with uncertainty. These scenarios are defined based on the probability distribution function of that parameter (in this paper, the normal distribution function is used for the electric load demand parameter). In order to account for load forecast errors, it is essential to incorporate load uncertainties into the model. To effectively model these uncertainties, the probability distribution function (PDF) of the load forecast error must be established. This paper assumes that the load forecast error follows a normal distribution, with the mean representing the forecasted peak load that the distribution company is required to meet. Fig. (8) illustrates an example of the continuous distribution function for the system's load forecast error, which has been discretized into seven intervals. Following the approach outlined in [21, 22], each interval corresponds to a width equal to one standard deviation of the load forecast error. The roulette wheel mechanism [23] is then employed to generate load scenarios based on varying levels of load forecasts and their associated probabilities derived from the aforementioned PDF. Initially, the range of [0,1] is populated with normalized load probabilities, as depicted in Fig. (9). Subsequently, a random number is generated within the same range. If this random number falls

within the normalized probability segment corresponding to a specific load forecast level on the roulette wheel (among the seven levels illustrated in Fig. (9)), that particular load forecast level is selected as a scenario by the roulette wheel mechanism. This process is repeated until a sufficient number of scenarios are generated. We refer to this combined approach of roulette-wheel selection and Monte Carlo simulation as RW/MCS.

5. NUMERICAL RESULTS

To demonstrate the effectiveness of the proposed model, it is applied to a sample distribution network shown in Fig. (10). The medium-voltage part of a 33 kV network is 54 nodes, which

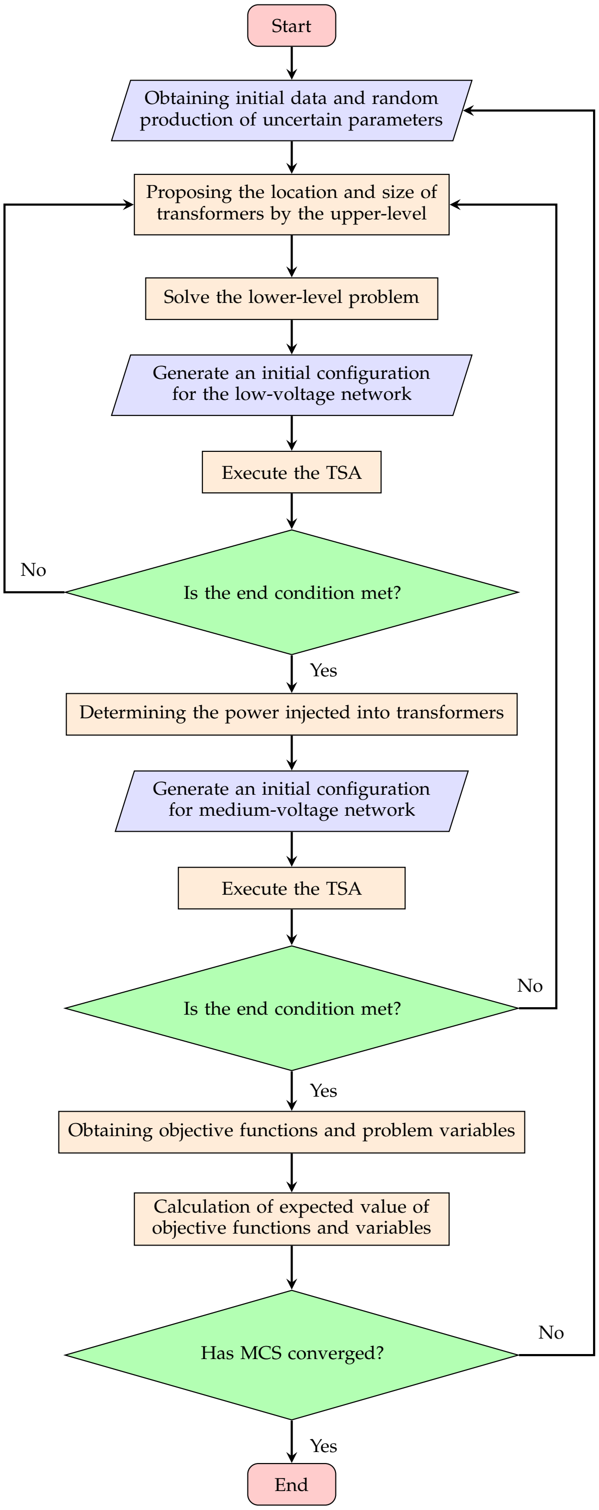


Fig. 7. Flowchart of the proposed approach

consists of 50 load points and is shown by solid circles with the specifications of Table (1). The low-voltage part is a 48-node 11 kV network, which consists of 48 load points, which are indicated by hollow white circles with specifications in Table (2).

Candidate points for installing power transformers are marked with T and blue color. The red dashed lines are candidates for installing lines on the low-voltage side. Tables (3)-(5) contain data about candidate lines, transformers, and distributed generation sources, respectively. All low-voltage network points are candidates for installing distributed generation sources. Also, the power factor ($\cos(\varphi)$) of all loads is 0.8. The maximum voltage setting for both medium-voltage and low-voltage networks is 10%. The load model is considered as a normal probability distribution function with the average values mentioned in Tables (1) and (2) and a standard deviation of 5%. The planning horizon of 5 years and the load duration curve are included in three load levels of 100%, 60% and 30% of peak demand. Discount rate is 10% and energy cost is 0.15 $/kWh. The penalty coefficients are 1000, 1500, 4000, 150, 100, and 1000 for $f_P V_{MV}$, $f_P I_{MV}$, $f_P S_{MV}$, $f_P V_{LV}$, $f_P I_{LV}$, and $f_P S_{LV}$, respectively.

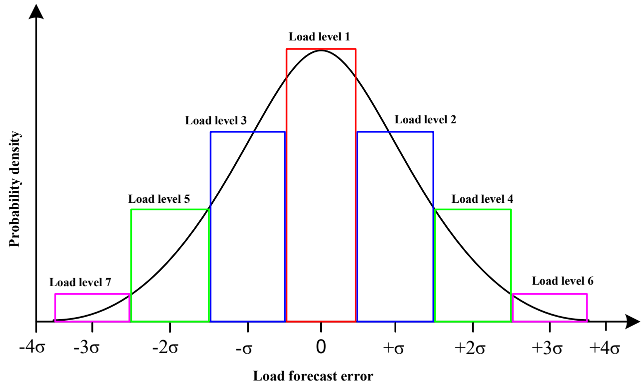


Fig. 8. A typical discretization of the continuous probability distribution of the load forecast error into 7 intervals

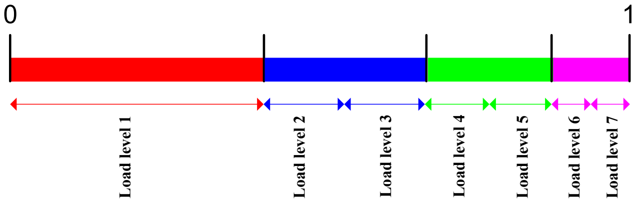


Fig. 9. Roulette wheel mechanism for the normalized probabilities of the load forecast levels

Table 1. The load points of medium-voltage network (kW)

Node	Load	Node	Load	Node	Load	Node	Load
1	700	14	700	27	500	40	400
2	600	15	800	28	700	41	500
3	500	16	900	29	800	42	400
4	500	17	700	30	900	43	500
5	600	18	500	31	700	44	600
6	700	19	600	32	600	45	800
7	500	20	800	33	500	46	800
8	900	21	800	34	400	47	400
9	500	22	600	35	900	48	800
10	900	23	400	36	300	49	500
11	300	24	500	37	800	50	800
12	400	25	900	38	700		
13	400	26	500	39	700		

The stop criterion of the used algorithm is 200 and the number

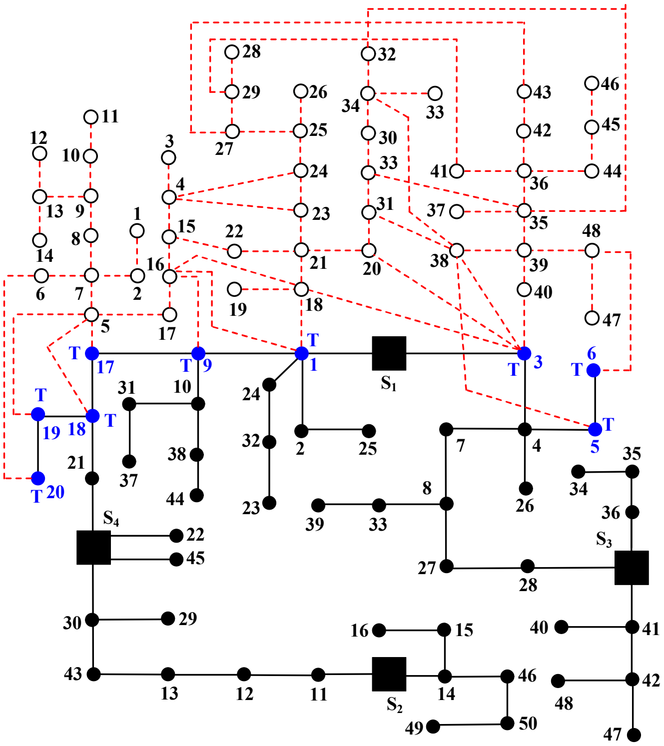


Fig. 10. The studied distribution network

Table 2. The load points of low-voltage network (kW)

Node	Load	Node	Load	Node	Load	Node	Load
1	620	13	700	25	600	37	600
2	640	14	600	26	400	38	250
3	770	15	450	27	740	39	400
4	520	16	770	28	400	40	350
5	620	17	750	29	400	41	580
6	400	18	750	30	400	42	790
7	600	19	600	31	700	43	700
8	600	20	600	32	750	44	680
9	640	21	720	33	720	45	600
10	400	22	700	34	740	46	680
11	530	23	700	35	750	47	620
12	700	24	680	36	600	48	620

of iterations in the MCS is 100. To validate the proposed model, three different scenarios are considered as follows:

- Scenario 1: Studying medium-voltage and low-voltage networks independently
- Scenario 2: using the model without considering DGs
- Scenario 3: applying the model by considering DGs

In all scenarios, it is assumed that the medium-voltage network is connected to the upstream network, so it has the necessary sufficiency to supply the load and the expansion of the network takes place only on the low-voltage side. In the first scenario, where the medium-voltage network and the low-voltage network are considered independently, the planning is such that first a list of transformers is proposed, then one of them is selected and the branch connected to it is identified, and then the operating conditions are checked. If the operating limits are not violated, the objective functions are evaluated and these steps are repeated until all the options in the list of connectable transformers are checked and the optimal solution of the objective functions is obtained. The second and third scenarios apply the proposed method, with the difference that in the third scenario, DGs are also applied. The DGs are modeled as PQ nodes.

Figs. (11)-(13) are the answers obtained from scenarios 1-3, respectively. In these figures, the connection points of transform-

Table 3. Data of the lines that can be installed in the low-voltage network

Type	R (Ω/km)	X (Ω/km)	Capacity (A)	Cost (1000 $/km)
1	0.7500	0.1764	61	17
2	0.4796	0.1673	84	22
3	0.3080	0.1596	114	30
4	0.1972	0.1496	156	42
5	0.1208	0.1442	208	54
6	0.0723	0.1262	303	85
7	0.0487	0.1217	400	125
8	0.0405	0.1196	453	140
9	0.0350	0.1180	500	165
10	0.0247	0.1140	645	220
11	0.019	0.11	700	270
12	0.017	0.09	850	310

Table 4. Transformers data

Type	Maximum capacity (MVA)	Cost (1000 $)
1	4	500
2	6	800
3	8	1000
4	10	1100

Table 5. DGs data

Type	Unit size (MVA)	Investment cost (1000 $)
1	1	500
2	1.5	450
3	1	400
4	0.8	470
5	1	800
6	1	800

ers are shown in blue color, the connection points of DGs are shown in pink color, and the connected lines are shown in red color. The numbers in parentheses also show the type of the desired element.

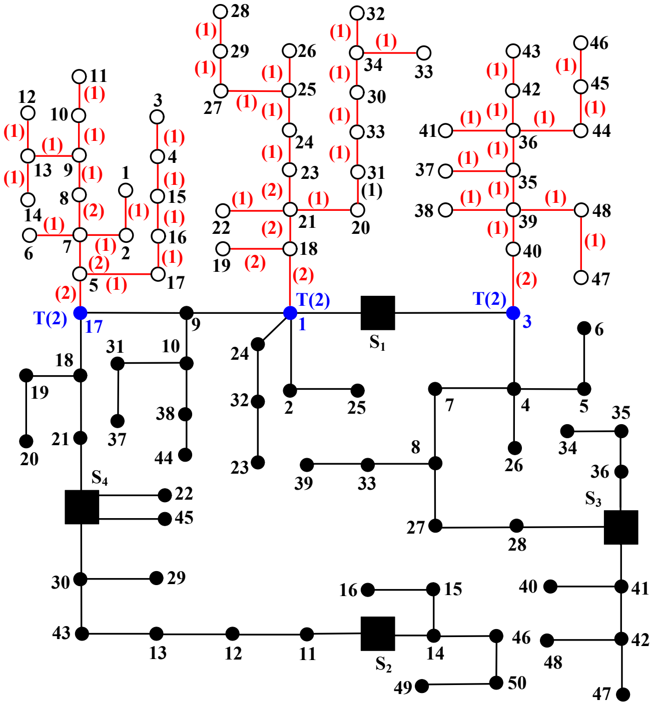


Fig. 11. Configuration resulting from the scenario 1

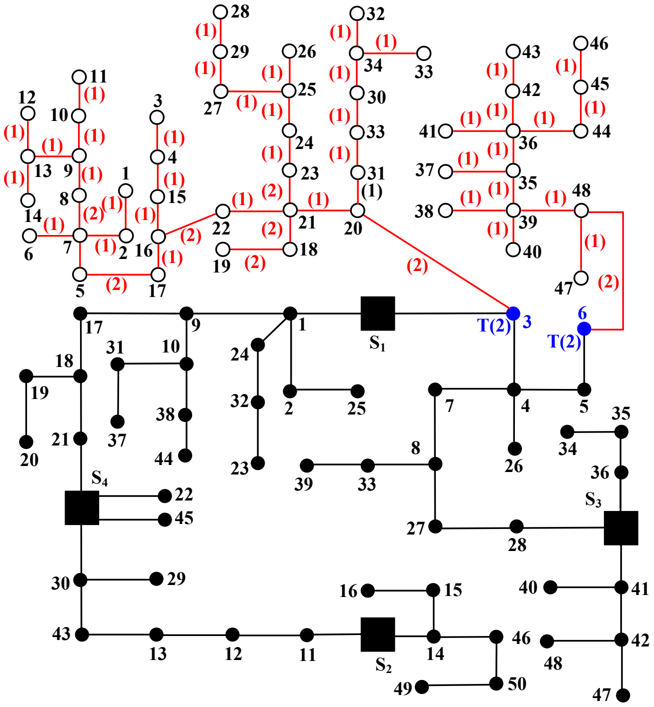


Fig. 12. Configuration resulting from the scenario 2

In Fig. (14), the amount of the objective function of the VSI of the medium-voltage network, which is the maximum value of VSI, is shown, and it can be seen that the VSI has increased significantly in the third scenario. Figs. (15) and (16) respectively show the voltage profile of the networks in different scenarios and it can be seen that the voltage profile has improved in the third scenario, which is actually a sign of reducing energy losses in this scenario. The consolidated results from Table (6) show that the total cost is lower in the second and third scenarios, which shows the advantages of using the method. The first scenario is a traditional method used by companies, and the other two scenarios use a bi-level model. The difference between these two cases is that the last case takes into account the penetration of the DGs in the low-voltage network.

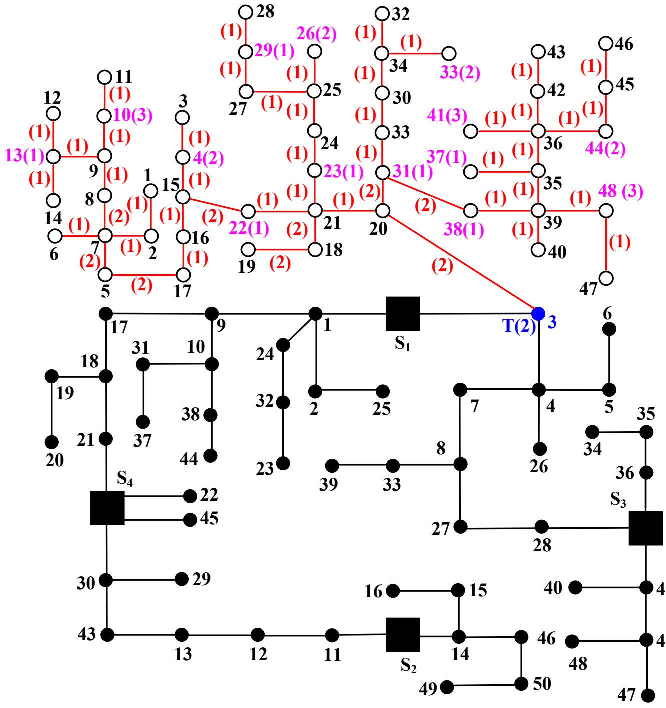


Fig. 13. Configuration resulting from the scenario 3

These three scenarios were applied to the same distribution system, where the results show that a bi-level integrated planning offers a lower cost compared to a traditional planning. According to Table (6), the cost of installing the transformer in the second and third scenario is reduced by 50% compared to the first scenario, the cost of the lines in the second and third scenario is reduced by 4.04% and 8.075% respectively compared to the first scenario. Losses in the second and third scenarios have decreased by 9.95% and 26.24% respectively compared to the first scenario, and the total cost has decreased by 4.33% and 6.17% in the second and third scenarios compared to the first scenario, respectively. The cost of lines, losses and total cost in the third scenario compared to the second scenario have decreased by 4.21%, 17.92% and 1.92%, respectively.

Electrical energy losses in the first scenario are higher compared to the second and third scenarios due to the higher current passing through the low-voltage network circuit. In other words, it can be said that the bi-level model reports topologies that have the lowest possible losses in the low-voltage network, while at the same time taking into account the requirements of the medium-voltage network (upstream network voltage stability), or it can be said that the distribution The electric current is bet-

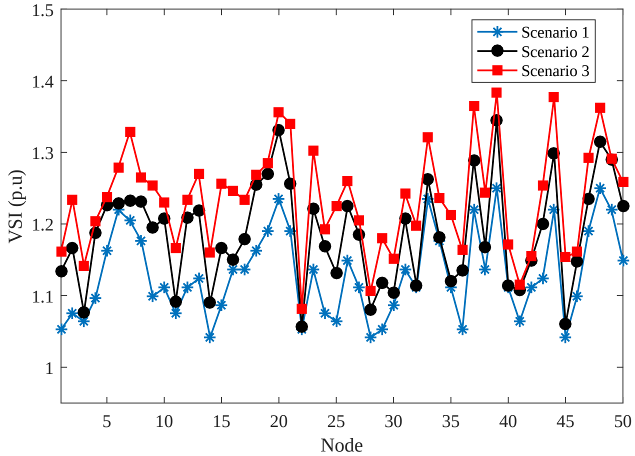


Fig. 14. Voltage stability index in different scenarios

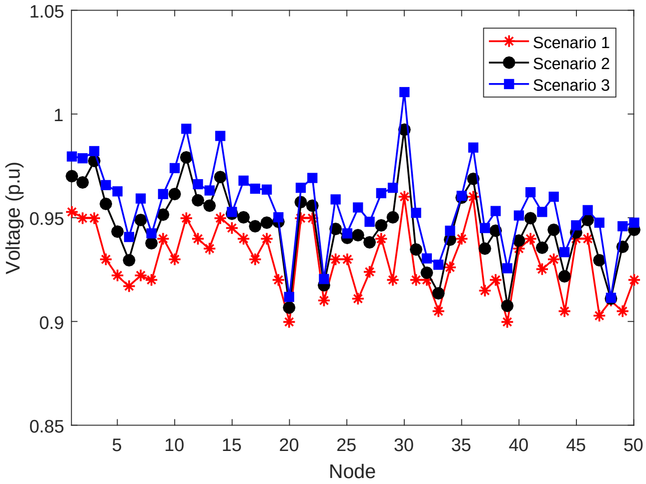


Fig. 15. Voltage of medium-voltage network nodes in different scenarios

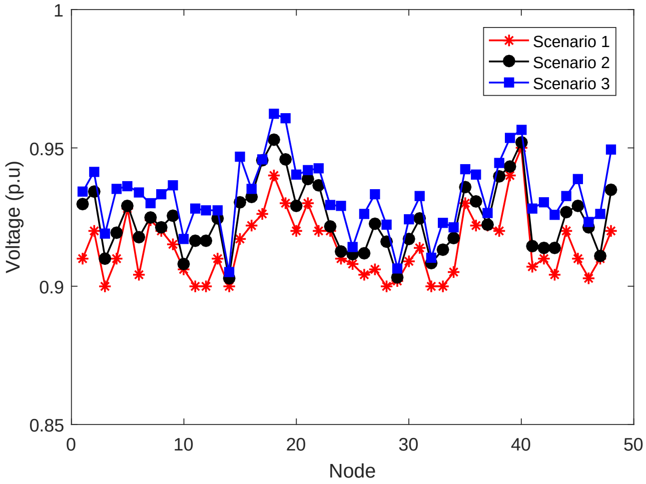


Fig. 16. Voltage of low-voltage network nodes in different scenarios

Table 6. Low-voltage network expansion costs (M$)

	Scenario 1	Scenario 2	Scenario 3
Transformer	1.76	0.88	0.88
DG	-	-	7.15
lines	314.32	301.63	288.94
Power losses	1.875	1.685	1.383
Total	317.955	304.195	298.353

ter in the proposed bi-level models. In the proposed bi-level model, it can be said that fewer transformers are used, which results in lower operating and investment costs. In the third scenario, due to the use of DG resources and their active power injection, the loss of electrical energy in the third scenario is less compared to the second scenario due to the reduction of the loading of lines and feeders. The topologies of the proposed lines in the second and third scenarios are almost similar, and the major difference is in the arrangement and type of distribution transformers. The important point in these results is that these calculations were done by considering the objective function used in the medium-voltage network, and if the results of the calculations in both networks were considered separately, perhaps better results would have been obtained, but from Since these two networks are connected, using the optimal solution in each independent network in the integrated network will increase losses, operating costs and investment (including increasing the number of transformers and lines). On the other hand, the results of improving the voltage stability index in the medium-voltage network also showed a relative improvement in the proposed bi-level model. To show the efficiency of the used solution method, the TSA method was compared with two approaches genetic algorithm (GA) and particle swarm optimization (PSO). As seen in Fig. (17), the speed of convergence and also finding a better solution, the TSA is better than GA and PSO. In this algorithm (TSA), it uses short-term memory to avoid falling into the local optimum. The task of this short-term memory is to store the latest movements, the number of which is limited, and these movements are kept in a list (forbidden list).

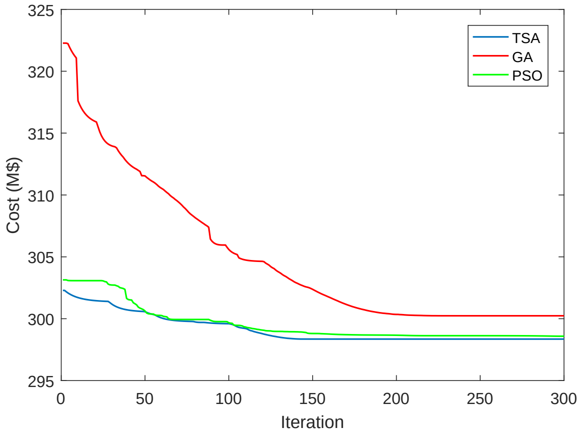


Fig. 17. Convergence and performance comparison of TSA, PSO, and GA for low-level objective function

6. CONCLUSION

In this paper, an integrated bi-level model for the distribution network is presented. In most of the studies, medium voltage and low voltage distribution networks are considered as two independent networks and separate planning has been done for each one; However, these two networks are not independent from each other and the effect of change in one network will definitely affect the other network. In this paper, medium voltage and low voltage distribution networks are included as a single system. The proposed model considers medium-voltage network at the upper-level and low-voltage network planning at the lower-level. The objective function at the upper-level is to maximize the voltage stability index and at the lower-level is to reduce the investment and operation costs, which are bound to a set of constraints. In addition, the influence of distributed generation sources is included in the low-voltage network. The proposed model takes advantage of the contradiction in planning for both networks, which is the placement and size of transformers, and in fact, this contradiction affects the currents circulating in both networks. When the bi-level model is solved, there is an influence on the related variables between both the upper and lower-level problems, which is provided by the transformers, because their capacity, location, and peak demand affect the solution of each level. This aspect provides the possibility of finding high-quality solutions and the results show the efficiency of the model and its solution method.

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