A Risk-based Two-stage Stochastic Optimal Power Flow Considering the Impact of Multiple Operational Uncertainties

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This paper shows an application of a scenario-based method for risk constrained stochastic optimal power flow (RC-SOPF) problem in electricity utilities. A two-stage stochastic programming framework is developed for dealing with various uncertainties. Customers' demand, wind power generation, and electricity price are considered as the uncertain parameters in the proposed RC-SOPF problem. The aim is to minimize the energy procurement costs, while preserving an acceptable risk level. The energy procurement cost consists of generators active power generation costs, cost of energy procurement from external network (e.g. pool market or upstream network) and operation maintenance cost of wind farms. To control the negative impacts of the uncertainties, variance and conditional value at risk (CVAR) are used as risk measures. The proposed model is implemented on the 39-bus New England test system. The obtained results show that CVAR is suitable index for management of the risk associated with uncertain parameters in comparison with variance.

keywords: *Risk constrained stochastic optimal power flow (RC-SOPF), scenario-based modeling, risk management, conditional value at risk (CVAR)*

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NOMENCLATURE

A. Sets:

SDC(i,t)SDC(i,t) SDC(i,t)SDC(i,t)

NB Set of system buses

NG Set of generating units

NP Set of buses connected to external network

NPQ Set of system PQ buses

NPV Set of system PV buses

NS Set of all scenarios

NW Set of wind farms

d_n Number of demand scenarios

w_n Number of wind scenarios

λ_n Number of electricity price scenarios

B. Indices:

b/j Index for system buses

i Index for conventional generators

s Index for scenarios

sl Index for slack bus

C. Parameters:

α Confidence level

β Weighting factor

$\zeta^w_{b,s}$ Percentage of realized wind power output at scenario s in bus i

π_λ Probability of demand scenario λ

π_d Probability of demand scenario d

π_s Probability of scenario s

π_w Probability of wind power generation scenario w

λ_D Demand price

$\lambda_{D,s}$ Demand price in scenario s

λ_P Pool market cost

$\lambda_{P,s}$ Pool market cost in scenario s

λ_W Operation& maintenance (OM) cost of the wind farm

$\lambda_{W,s}$ OM cost of the wind farm in scenario

a_i, b_i, c_i the quadratic cost coefficients of thermal unit i

$cos(\varphi_{lag,b}/cos(\varphi_{lead,b}$ Lag/lead power factor limits of the wind farms located at node b

$(P)^D_{d,max/min}$ Max/minimum active power consumption of load at scenario d

$(P/Q)^D_{d,max/min}$ Max/minimum active/reactive power consumption of load connected to bus b

$(P/Q)^G_{d,max/min}$ Max/minimum active/reactive power of generator i

$(P/Q)^P_{d,max/min}$ Max/minimum active/reactive power of pool market to bus b

$(P/Q)^W_{d,max/min}$ Max/minimum active/reactive power of wind power to bus b

P^W_{avl} Available wind power generation

$P^W_{b,r}$ Wind farm rated capacity installed in bus b

P^W_r Wind farm rated capacity

$V_{b,max/min}$ Max/minimum value for voltage magnitude of the b-th bus

v^c_{in}/v^c_{out} Cut-in/out speed of wind turbine in m/s

v_{rated} Rated speed of wind turbine in m/s

V_w Wind speed in m/s

$S_{bj,max}$ Maximum power flowing through the line

Y_{bj}/γ_{bj} Admittance magnitude/angle of line

D. Variables:

∇_s A continuous non-negative variable

$CVaR$ Conditional value at risk (risk management index)

VaR Value at risk (risk management index)

$Variance$ Variance (risk management index)

RP Resource problem

EEV Expected outcome of using the expected value

VSS Value of stochastic solution

$P^D_{b,s}/Q^D_{b,s}$ Active/reactive power consumption of load connected to bus b in scenario s

$P^G_{sl,s}$ Active power production of slack bus in scenario s

$P^G_{i,s}/Q^G_{i,s}$ Active/reactive power production of generator thermal unit i in scenario s

$P^P_{b,s}/Q^P_{b,s}$ Active/reactive power pool market to bus b in scenario s

$P^W_{b,s}/Q^W_{b,s}$ Active/reactive wind power production injected to bus b in scenario s

$\bar{x}$ Vector of dependent variable

$\bar{x}_s$ Vector of dependent variable in scenario

$\bar{U}$ Vector of control variable

$\bar{U}_s$ Vector of control variable in scenario

S_{bj} Power flowing through the line

V_b/θ_b Voltage magnitude/angle of bus

$V_{b,s}/\theta_{b,s}$ Voltage magnitude/angle of bus in scenario

1. INTRODUCTION

A. Background and motivations

Utilization of renewable energy sources (RES) has significantly grown worldwide, because of increasing energy consumption, environmental concerns, and decreasing fossil fuel resources [1]. Due to the increasing usage of wind turbines, power systems will surely face with risk, because wind power is a volatile and intermittent source of RES.

Energy management in the presence of RES, includes economic dispatch (ED), unit commitment (UC), and optimal power flow (OPF) problem which have been extensively studied recently [2]. The electricity market consists of different entities which have different goals and motives for demand response in each of them. Among different institutions, retailers have highest interaction with electrical energy consumers. Therefore, retailers role are to buy the electrical energy from wholesale markets and sell it to consumers. It is clear that the aim of a retailer is to achieve maximum profit. Thus, the retailer should consider the risk of profit reduction due to the uncertainty of demand and wind power generation, as well as electricity prices. In this paper, a risk constrained stochastic optimal power flow problems (RC-SOPF) is proposed to determine the optimal strategy of energy procurement from various energy resources.

The wind power generation, energy price and demand uncertainties make the operation of system difficult and risky task. Cost calculation considering the associated risks has financial benefits.

B. Literature review

The previous studies in OPF problem can be categorized as follows.

- Deterministic OPF problem solved by heuristic algorithms [3–8]
- Probabilistic OPF problem solved by probabilistic methods (stochastic programming [9–11], point estimate method [12, 13], robust optimization [14], information gap decision theory [15])
- Probabilistic risk-based OPF problem [22–24]

To solve this optimization problem some heuristic algorithms are introduced in recent years, such as modified cuckoo search [3], non-dominated sorting multi objective gravitational search algorithm (NSMOGSA) [4], improved Group Search Optimization (IGSO) [5], Multi-Objective Modified Imperialist Competitive Algorithm (MOMICA) [6], chaotic artificial bee colony (CABC), particle swarm optimization with an aging leader and challengers (ALC-PSO) [8]. Ref [16] proposed a multi-objective optimal power flow model of multiple-energy system. This model helps to achieve more comprehensive and efficient use of energy and multiple-energy input to reduce costs and it is solved by non-dominated sorting genetic algorithm (NSGA-II) and maximum satisfaction method.

There are different methodologies for handling uncertainties in OPF problem [17]. A stochastic multi-period OPF problem which includes an offshore wind farm connected to the grid by a line-commutated converter high-voltage dc link is proposed in [9]. In [18] DC security constrained optimal power flow (SCOPF) problem is formulated chance constraints and it achieves a good tradeoff between security and cost. In ref [19], the alternating current optimal power flow (ACOPF) problem is formulated as a nonconvex quadratically-constrained quadratic program (QCQP) with complex variables. A two-level hierarchical dynamic stochastic optimal power flow (DSOPF) control structure for scaling up the DSOPF control scheme for large power systems is proposed in [10]. Stochastic security-constrained optimal power flow (SSCOPF) problem is presented and handled in [11] by considering uncertainties. Point estimate method (PEM) [13], Quasi Monte Carlo simulation (QMCS) and Latin hypercube sampling (LHS) [12], are used to compare the solution of probabilistic OPF problem. An adjustable robust optimization approach is presented in [14] to account for the uncertainty of renewable energy sources (RES) in OPF problem. In [20],chance constrained optimal power flow (CC-OPF) problem is formulated based on procure minimum cost energy, generator reserves, and load reserves given uncertainty in renewable energy production, load consumption, and load reserve capacities. In [15] information gap decision theory (IGDT) is utilized for the uncertainties handling at the presence of offshore wind farms. Authors in [21] reduced the inaccuracy of assuming Gaussian distributions for wind forecast errors, based on RCC OPF formulation. The RCC OPF formulation accounts for uncertainty in the parameters of statistical models that describe the deviations of wind generation from its forecast.

The chance constrained programming is modeled for solution of risk-based OPF problem in [22] and [23]. Value at risk (VaR) and conditional value at risk (CVaR) indices are utilized in [24] for modeling risk in microgrid. In [25], wind farm and plug-in electric vehicles (PEVs) are added to stochastic OPF problem and Gbest guided artificial bee colony algorithm (GABC) is used for dealing with the optimization problem.

In recent years, some new indices such as value at risk (VaR) and conditional value at risk (CVaR) have been developed for risk measuring in finance and economy fields. As indicated in [26–28], the CVaR index is an appropriate choice for stochastic programming. Therefore, it is applied to OPF problem for minimization of total cost of energy hub in [29]. Also, VaR is the maximum loss not exceeded with a given probability defined as the confidence level [30]. In this study, variances and CVaR indices are employed to deal with the economic risk due to the uncertain behaviors of wind power generation, demand of electrical power and price of energy.

Table 1. Comparison of different models in the literature with the proposed RC-SOPF

Ref	OPF is solved?		Uncertainty modeled in OPF?			WFs O&M cost considered?	Stochastic programming?	Risk management is performed?	
	In transmission networks	In microgrid	Demand	Wind power	Price			Variance?	CVar?
[3–8]	Y	N	N	N	N	N	N	N	N
[9,10]	Y	N	N	Y	N	N	Y	N	N
[11]	Y	N	N	N	N	N	Y	N	N
[12–15]	Y	N	N	Y	N	N	N	N	N
[22,23]	Y	N	N	Y	N	N	N	N	N
[24]	N	Y	Y	Y	N	N	Y	N	N
[29]	Y	N	Y	Y	Y	N	Y	N	Y
Proposed	Y	Y	Y	Y	Y	Y	Y	Y	Y

For Doubly fed induction generators (DFIGs) and permanent magnet synchronous generators (PMSGs) [31], [15] technologies, the operation maintenance (OM) cost of wind farm is typically around 20-25% of the total lifetime costs of the installation [32]. This amount of cost is really significant for system operators, such that some utilities planned to reduce it until 2020 by 35% [33–35]. Thu it is necessary to consider this component of wind power generation cost in the network operator decision making process.

C. Contributions

This paper is focused on solving RC-SOPF problem using concept of risk management to achieve minimizing the energy procurement costs. The cost is calculated at three different cases: without considering risk management, considering risk management with variance index and considering risk management with CVAR index. The RC-SOPF is modeled as a nonlinear programming (NLP) optimization problem and solved using the SNOPT solver [36] in GAMS environment [37]. The obtained result is shown that considering risk management is suitable method to achieve minimum costs with an acceptable level of risk. Hence, the main contributions of this work are summarized as follows:

1) Stochastic behavior of load demand, wind power generation and electricity price are considered in the RC-SOPF problem via scenario based modeling approach.

2) To the best of the authors' knowledge, no work in the existing literature includes the OM cost of wind farms in the RC-SOPF problem. In this work, the OM cost of wind power generation is also considered in the proposed RC-SOPF problem model.

3) Risk is measured via CVaR and variance indices in transmission networks.

In summary, based on the aforementioned literature survey and the above contributions, the knowledge gap covered in this paper is given in Table 1.

D. Paper organization

This paper is organized as follows: the scenario based uncertainty modeling is discussed in Section 2. In Sections 3 the formulation of proposed RC-SOPF problem is presented. The obtained results on the 39-bus New England system are given and discussed in Section 4. Finally, Section5 summarizes the findings and concludes the paper.

2. SCENARIO BASED UNCERTAINTY MODELING

A. Modeling of load uncertainty

Due to the stochastic nature of load, it is necessary to model the load uncertainty in both planning and operation of power systems [38]. Generally, load uncertainly can be characterized via normal or Gaussian probability density function (PDF) [39,40].

Table 2. Wind-load- electricity price scenarios and their probabilities

			Scenario number	Load (%)	λ_{pool}	Wind (%)	π_s
Load scenarios			S_1	0.97	44	1	0.003
	Load (%)	π_d	S_2	0.97	44	0.5	0.024
d_1	97	0.15	S_3	0.97	44	0	0.003
d_2	100	0.7	S_4	0.97	46	1	0.009
d_3	103	0.15	S_5	0.97	46	0.5	0.072
			S_6	0.97	46	0	0.009
Wind power generation scenarios			S_7	0.97	50	1	0.003
	Wind (%)	π_w	S_8	0.97	50	0.5	0.024
w_1	0	0.1	S_9	0.97	50	0	0.003
w_2	50	0.8	S_{10}	1	44	1	0.014
w_3	100	0.1	S_{11}	1	44	0.5	0.112
			S_{12}	1	44	0	0.014
Pool price			S_{13}	1	46	1	0.042
	λ_{p}	π_{λ}	S_{14}	1	46	0.5	0.336
p_1	44	0.2	S_{15}	1	46	0	0.042
p_2	46	0.6	S_{16}	1	50	1	0.014
p_3	50	0.2	S_{17}	1	50	0.5	0.112
			S_{18}	1	50	0	0.014
			S_{19}	1.03	44	1	0.003
			S_{20}	1.03	44	0.5	0.024
			S_{21}	1.03	44	0	0.003
			S_{22}	1.03	46	1	0.009
			S_{23}	1.03	46	0.5	0.072
			S_{24}	1.03	46	0	0.009
			S_{25}	1.03	50	1	0.003
			S_{26}	1.03	50	0.5	0.024
			S_{27}	1.03	50	0	0.003

According to Fig.1, the values of mean and variance of the uncertain parameter are needed in the normal PDF. In this paper, it is assumed that the values of mean and standard deviation of the load PDF, i.e. μ_D and σ_D are known. In scenario based approach, the entire interval of load variations is divided into some distinct subintervals as it can be seen from Fig. 1. For each subinterval, proper mean and probability are calculated. In this paper, three scenarios are considered for characterizing load uncertainty, and hence the corresponding normal PDF is divided into 3 distinct subintervals. The corresponding mean and probabilities of these subintervals are calculated by (1) and (2). The index π_d is used for probability of d-th load scenario. Moreover, It is noticeable that $P^D_{d,min}$ and $P^D_{d,max}$ are minimum and maximum boundaries of d-th load subinterval respectively, as it is shown in Fig. 1. The obtained load scenarios and corresponding probabilities are given in Table 2.

$$\pi_d = \int_{P^D_{d,min}}^{P^D_{d,max}} 1\sqrt{2\pi\sigma_D^2}exp[-\frac{(P_D-\mu_D)^2}{2\sigma_D^2}dP_D \quad (1)$$

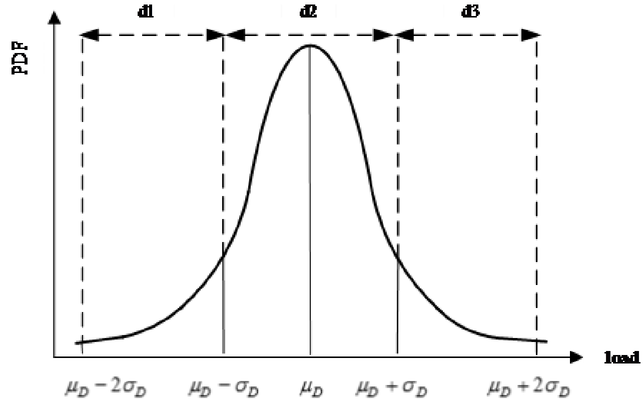

Fig. 1. The load PDF and load uncertainty scenario generation

$$P_d^D = \frac{1}{\pi_d} \times \int_{P^D_{d,min}}^{P^D_{d,max}} [P_D \times \frac{1}{\sqrt{2\pi\sigma_D^2}}exp[-\frac{(P_D-\mu_D)^2}{2\sigma_D^2}]]dP_D \quad (2)$$

B. Wind power generation uncertainty modeling

Different methods have been used to deal with the wind speed uncertainty, which are conformed to the Weibull PDF with diurnal pattern and autocorrelation factor [29,39,41]. The Rayleigh PDF of the wind speed is one of these methods which can be shown as follows.

$$PDF(v) = (\frac{v}{c^2})exp[-(\frac{v}{\sqrt{2}c})^2] \quad (3)$$

Figure 2 shows the Rayleigh PDF for c=11.28 [29]. In scenario based modeling, the wind speed entire variation range is divided into some intervals, which are called scenarios. Following equations are used for calculation of probability of each scenario w and corresponding mean value wind speed v_w , respectively.

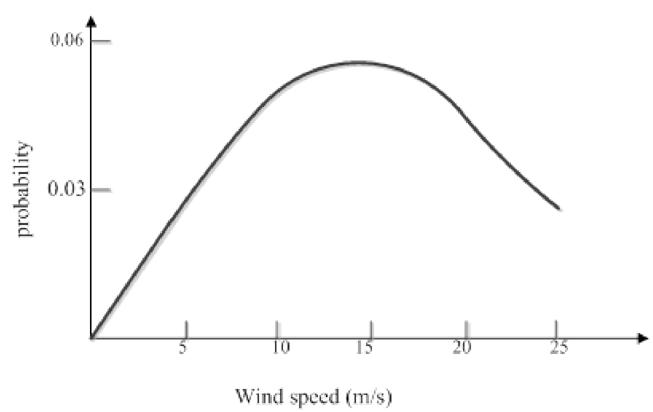

Fig. 2. A typical wind speed distribution expressed by Rayleigh PDF

$$\pi_w = \int_{v_{i,w}}^{v_{f,w}} (\frac{v}{c^2})exp[-(\frac{v}{\sqrt{2}c})^2]dv \quad (4)$$

$$v_w = \frac{1}{\pi_w} \times \int_{v_{i,w}}^{v_{f,w}} \left(v \times (\frac{v}{c^2})exp\left[-(\frac{v}{\sqrt{2}c})^2\right]\right)dv \quad (5)$$

Where, v_w is the wind speed at -th wind scenario $v_{i,w}$ is the starting point and $v_{f,w}$ is ending points of wind speed's interval at -th scenario. Also, c is scaling parameter which is calculated by recorded historical wind data. The characteristic curve of a wind turbine shows the relation between the available wind speed and the generated wind power. In this study, a linear characteristic curve is used for modeling generated power from wind turbine which is expressed by Fig. 3. From this curve, the output power of wind turbine versus the wind speed is formulated as follows.

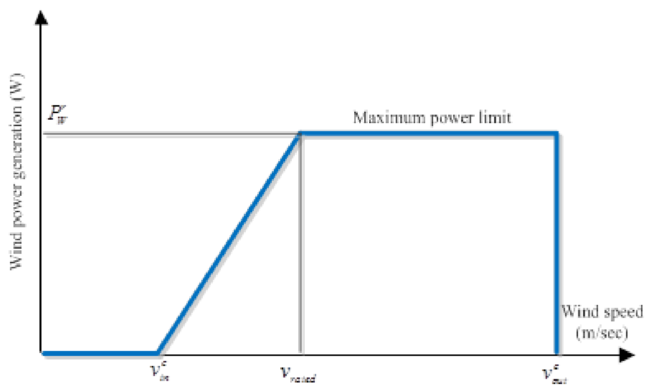

Fig. 3. The power curve of a wind turbine

$$P^w_{avl} = \begin{cases} 0 & v_w \le v^c_{in} \ or \ v_w \ge v^c_{out} \\ \frac{v_w - v^c_{in}}{v_{rated} - v^c_{in}} P^w_r & v^c_{in} \le v_w \le v_{rated} \\ P^w_r & v_{rated} \le v_w \le v^c_{out} \end{cases} \quad (6)$$

C. Energy price uncertainty modeling

In the recent years, the electricity industry adopts a market structure to respond to electricity demand with a high reliability at minimum cost. Thus, producers are in competition with each other for selling electric energy to consumers with huge benefits [42].

The electricity market acts as an interface among the production, transmission, distribution, and units' control. So the economic structure of the electricity industry is divided into four parts: wholesaler, transmission, major shopping, and retailer. In the wholesale market, owners of power plants provide electricity in the market. The wholesale markets authorize transactions among producers, retailers, and other intermediaries for delivery of electricity in short time, or for the future delivery. Therefore, retail market is as one part of the electric market allows retailers to act as intermediaries to purchase consumers electricity demand and provide their consumers. The market has many customers which allows them to choose their electricity supplier.

The pool market is defined as a concentrated market which is prepared for purchaser and consumer. The retailers and consumers order their amount of electrical power producer and demand to pool market for exchanging the electricity of energy. The pool price of a particular electricity market is a random variable. According to the distribution of pool price which is expressed in [43], three scenarios are considered for pool price and presented in Table 2. Besides ,the characteristics of three wind power generation scenarios are summarized in Table 2 [39].

It is assumed that the individual scenarios of power demand, electricity price, and wind power generation are independent. Thus, these scenarios are combined to construct the whole set of scenarios. Consequently, the probability of each combined scenario is calculated as follows.

$$\pi_s = \pi_d \times \pi_\lambda \times \pi_w \quad (7)$$

Where π_d, π_λ and π_w are the probabilities of the d-th demand, λ-th electricity prices and the w-th wind scenarios, respectively. The total number of scenarios, i.e., , will be $d_n \times \lambda_n \times w_n$, where d_n, λ_n, w_n are the number of demand , electricity prices and wind states.

D. Risk indices in the stochastic programming

Risk means the uncertainty to the future results [44]. IEEE defines "risk" as the result of probability and consequence [44]. The risk management in the electricity market is an analysis tool to participate in the electricity market process which provides the decision preferences through risk identification, measurement, analysis, and other aspects [44]. In the scenario-based models, there are various methods for quantifying risk impacts, such as variance, shortfall probability, expected shortage, value-at-Risk (VaR), Conditional Value-at-Risk (CVaR) [44]. In this paper, variance and CVaR indexes are used for measurement of risk.

D.1. Variance

The variance index can be characterized by two parameters: the expected return and the variance of this return [43]. By considering the objective function with Probability for scenario s, the variance is defined as follows:

$$Variance(x) = \sum_s \pi_s \left[f(x,s) - \sum_s \pi_s f(x,s) \right]^2 \quad (8)$$

D.2. CVaR

The CVaR is the expected value, not exceeding Value-at-risk (VaR) [29]. The CVaR gives the frequency of extreme events and the severity of losses in the case of undesired events, but the VaR only gives the frequency of extreme events [29]. In optimization problems where the objective function is minimized, the CVaR is defined as follows.

$$CVaR = VaR + \frac{\sum \pi_s \Delta_s}{1-\alpha} \quad (9)$$

$$f(x,s) - VaR \le \Delta_s \quad (10)$$

$$\Delta_s \ge 0 \quad (11)$$

where CVaR is computed as the expected objective function in the $(1\alpha) \times 100\%$ worst scenarios. α is indicating the upper tail of the cost distribution in minimization problem.

3. PROBLEM FORMULATION

In this section the description of two stage stochastic optimization model along with the considered objective functions and the problem constraints are described.

A. Two stage stochastic optimization framework

In this paper, a two-stage stochastic optimization model is developed. In this approach, the decision variables are divided into two types. The first type variables are called “here and now” or "first stage" control variable. These variables remain the same for all possible scenarios. The second type are called “wait and see” or "second stage" variables which are determined specifically for each scenarios [29,45].

The SOPF problem variables subsets can be stated as follows.

$$\begin{aligned}\bar{u} &= \begin{bmatrix} V_b & ,\forall b \in NG & \\ P^W_{b,s} & ,\forall b \in NW & ,\forall s \\ Q^W_{b,s} & ,\forall b \in NW & ,\forall s \\ P^P_{b,s} & ,\forall b \in NP & ,\forall s \\ Q^P_{b,s} & ,\forall b \in NP & ,\forall s \end{bmatrix} \\ \bar{x} &= \begin{bmatrix} V_{b,s} & ,\forall b \in NPQ & ,\forall s \\ \theta_{b,s} & ,\forall b \in NB & ,\forall s \\ S_{bj,s} & ,\forall b \in NB & ,\forall s \\ Q^G_{i,s} & ,\forall i \in NG & ,\forall s \\ P^G_{i,s} & ,\forall i \in sl & ,\forall s \end{bmatrix}\end{aligned} \tag{12}$$

where, $\bar{U}$ is the set of control variables, $\bar{X}$ is the set of state variables. As it is aforementioned, the set of control variables is also divided into two distinct subsets, namely here and now and wait and see decision variables. The set of here and now decision variables (D_{HN}) are as follows:

$$D_{HN} = \begin{Bmatrix} V_b & ,\forall b \in NG \\ P^G_{i,s} & ,\forall b \in NG \end{Bmatrix} \tag{13}$$

Also, the set of wait and see decision variable (D_{WS}) are as follows.

$$D_{WS} = \begin{Bmatrix} P^W_{b,s} & ,\forall b \in NW & ,\forall s \\ Q^W_{b,s} & ,\forall b \in NW & ,\forall s \\ P^P_{b,s} & ,\forall b \in NP & ,\forall s \\ Q^P_{b,s} & ,\forall b \in NP & ,\forall s \end{Bmatrix} \tag{14}$$

B. Stochastic optimal power flow

B.1. Objective function

Account for uncertainties inherent to power systems, stochastic techniques have been used since the early seventies, where the uncertainty in system demand was first considered in a standard power flow problem [46]. Optimal Power Flow (OPF) is a nonlinear optimization to determine the optimal amount of electrical variables in a power system, with an objective function and some constraints. These constraints are included power flow equations, supply and demand bid limits, power transmission limits (security limits), line thermal limits, reactive generator limits and voltage limits. In this paper, the objective function is to minimize the energy procurement costs. SOPF statistical characteristics of the output variables, such as: bus voltages and angles, active and reactive powers, and price that scenario-based method is used to find the statistical characteristics of the output variables.

In this study the loads, wind power generation, and the price of energy exchange with pool market (or external network) are considered as three uncertain parameters [46]. In this paper the overall cost function is composed of three terms (i.e. system generators cost, pool market cost and OM cost of wind farm). The cost function in scenario s can be mathematically expressed as follows:

$$\cos t(\bar{u}_s, \bar{x}_s) = GC_s + PC_s + WC_s \tag{15}$$

In (15), the generation cost of thermal units is defined as

$$GC_s = \sum_{i=1}^{NG} F(P^G_{i,s}) \tag{16}$$

where

$$F(P^G_{i,s}) = a_i(P^G_{i,s})^2 + b_i P^G_{i,s} + c_i \tag{17}$$

The second term of (15) is pool market cost which defined as:

$$PC_s = \sum_{b=1}^{NP} \lambda_{P,s} P^P_{b,s} \tag{18}$$

Based on IRENA report [47], the OM cost of wind turbine is expressed as follows:

$$WC_s = \sum_{b=1}^{NW} \lambda_{W,s} P^W_{b,s} \tag{19}$$

where, λ_{w_i} is the OM cost of i-th wind farm. Therefore with considering scenario probabilities, the expected objective function of proposed problem is calculated as follows.

$$F = \sum_{s=1}^{NS} (\pi_s \times \cos t(\bar{u}_s, \bar{x}_s)) \tag{20}$$

B.2. Problem constraints

Equality constraints

The obtained solution should satisfy the power balance constraints which are described mathematically as follows [48].

$$P^G_{b,s} + P^P_{b,s} + P^W_{b,s} - P^D_{b,s} = V_{b,s} \sum_{j=1}^{NB} V_{j,s} Y_{bj} \cos(\theta_{b,s} - \theta_{j,s} - \gamma_{bj}) \tag{21}$$

$$Q^G_{b,s} + Q^P_{b,s} + Q^W_{b,s} - Q^D_{b,s} = V_{b,s} \sum_{j=1}^{NB} V_{j,s} Y_{bj} \sin(\theta_{b,s} - \theta_{j,s} - \gamma_{bj}) \tag{22}$$

Inequality constraints

The active power, reactive power generation of the generators and voltage of buses should be in the allowed range as follows:

$$P^G_{i,\min} \le P^G_{i,s} \le P^G_{i,\max} \qquad ,\forall i \in NG, \quad \forall s \tag{23}$$

$$Q^G_{i,\min} \le Q^G_{i,s} \le Q^G_{i,\max} \qquad ,\forall i \in NG, \quad \forall s \tag{24}$$

$$V_{b,\min} \le V_{b,s} \le V_{b,\max} \qquad ,\forall i \in NB, \quad \forall s \tag{25}$$

The power transmitted from the branches is constrained to its maximum value as follows.

$$0 \le S_{bj} \le S_{bj,\max} \qquad ,\forall b \in NB, \quad \forall s \tag{26}$$

The active/reactive power outputs of pool market are limited as follows.

$$P^P_{b,\min} \le P^P_{b,s} \le P^P_{b,\max} \qquad ,\forall b \in NP, \quad \forall s \tag{27}$$

$$Q^P_{b,\min} \le Q^P_{b,s} \le Q^P_{b,\max} \qquad ,\forall b \in NP, \quad \forall s \tag{28}$$

Also the following limits are considered for the available active/reactive power of wind farms.

$$0 \le P^W_{b,s} \le \zeta^W_{b,s} \times P^W_{b,r} \qquad ,\forall b \in NW, \quad \forall s \tag{29}$$

$$Q^W_{b,\min} \le Q^W_{b,s} \le Q^W_{b,\max} \qquad ,\forall b \in NW, \quad \forall s \tag{30}$$

In this paper, the reactive power output of wind farms are limited to the corresponding active power output as follows.

$$Q^W_{b,\max} = tg(\varphi_{lag}) \times P^W_{b,s} \tag{31}$$

$$Q^W_{b,\min} = -tg(\varphi_{lead}) \times P^W_{b,s} \tag{32}$$

C. Formulation of risk

SOPF problem can be solved to minimize the energy procurement costs, by considering the risk level. Therefore in RC-SOPF, risk level is considered as the second objective function. Variance and CVAR indices are utilized in this paper for modeling risk.

The objective functions in scenario-based method for two different cases are defined as follows:

• Objective function without risk management

$$OF = \min((1-\beta) \times F) \tag{33}$$

• Objective function by considering risk management with calculation of variance

$$OF = \min\{(1-\beta) \times Variance(x) + \beta \times F\} \tag{34}$$

• Objective function by considering risk management with calculation of CVaR

$$OF = \min\{(1-\beta) \times CVaR + \beta \times F\} \tag{35}$$

The flowchart of the method proposed in this paper is showing in Fig.4.

4. SIMULATION RESULTS

In this article, the proposed RC-SOPF problem is formulated as nonlinear programing (NLP) problem and it is implemented by using Generalized Algebraic Modeling Systems (GAMS) software [37] and solved by SNOPT [36] solver. In this paper, modified 39-Bus New England system is studied, as it is shown in Fig. 5. A wind farm is added to bus 16 rated as 600MW [49] and a pool market is added to bus 25. The OM cost of this wind farm is assumed to be 17.5 \$/MWh, which is adopted from [47]. Bus 31 assumed as slack bus [50], 30, 32,33,34,35,36,37,38 and 39 are taken as PV buses and the remaining 29 buses are PQ buses.

The system data which include the active/reactive loads, under load tap changing transformers (ULTCs) values, transmission system parameters, reactive power limits of generators along with voltage limits are given in [50]. In this paper, in order to investigate the impact of risk indices on the SOPF problem, three different cases are studied as follows.

• Case-I: Solving RC-SOPF without considering risk management

• Case-II: Solving RC-SOPF considering risk management by variance calculation

• Case-III: Solving RC-SOPF considering risk management by calculation

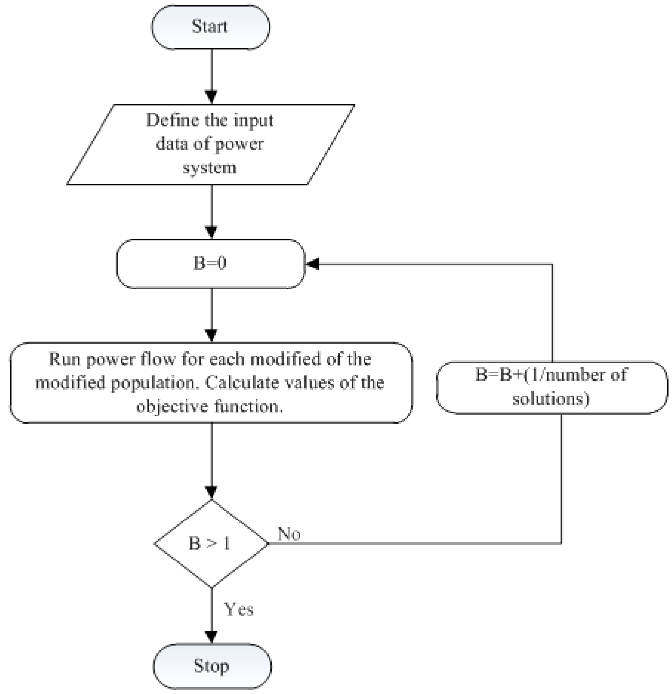

Fig. 4. Flowchart of the method proposed

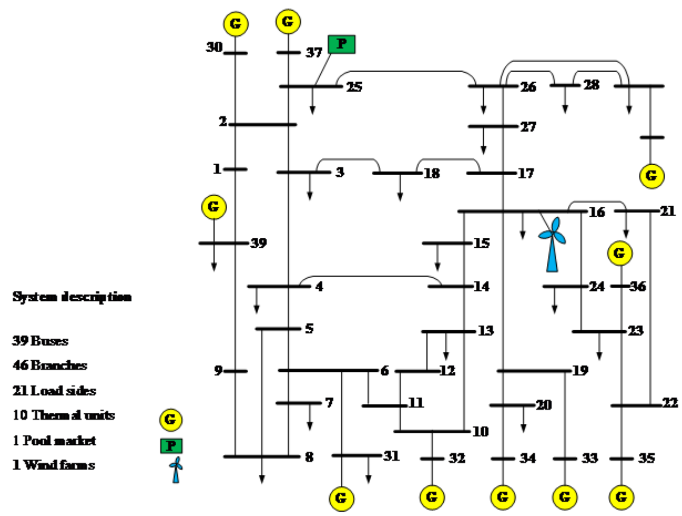

Fig. 5. Single line diagram of 39-Bus New England system

A. Case-I: Solution of SOPF without considering risk indices

In this part, objective function is neglected the risk and consequently the total system cost minimized. The computational time of about 428 sec. on a PC with Intel Core i5 CPU@2.53 GHz and 8G RAM. The energy procurement cost is 147455.4\$/h. For this solution, active power generation in slack bus in all scenarios is shown in Fig.6. Additionally, active and reactive purchased power from pool market in all scenarios are presented in Fig.7 and active and reactive power output of wind farm in all scenarios for this case are supposed in Fig. 8.

B. Case-II: Considering risk via variance index

The goals include in risk aversion, the minimum risk and minimize the cost, simultaneously. The variance of the cost is calcu-

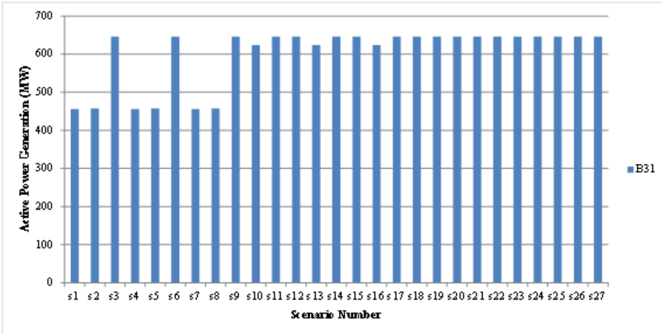

Fig. 6. Active power generation in slack bus (i.e. bus 31) in all scenarios (in MW) in Case-I.

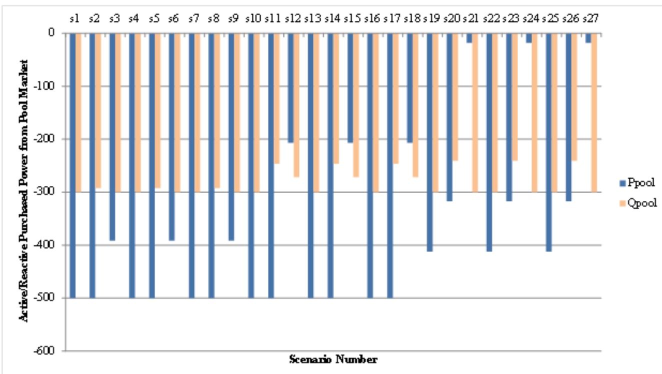

Fig. 7. Active and reactive purchased power from pool market (located at bus 25) in all scenarios (in MW and MVAR) in Case-I.

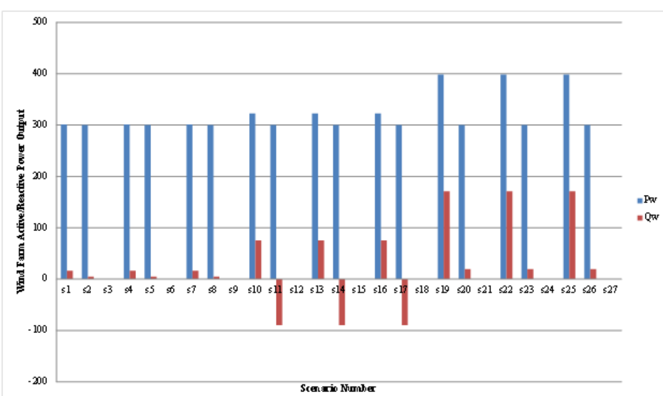

Fig. 8. Active and reactive power output of wind farm (located at bus 16) in all scenarios (in MW and MVAR) in Case-I.

lated for $\beta = [0, 1]$ in 20 solutions. The computational time is 985 sec. Pareto optimal cost front of variance is depicted in Fig.9. As it is expected, the variance of the cost will be decreasing while the value of β decreases. The values of the cost and variance for all 20 Pareto optimal solutions are demonstrated in Table 3. Among these optimal solutions, for $\beta = 0$, the maximum cost has been achieved, with the total cost equal to 229228.2426 \$/h and the variance is equal to 64539.9540 \$/h2. For this solution, active power generation in slack bus for $\beta = 0$ in all scenarios is given in Fig.10. Additionally, active and reactive purchased power from pool market for $\beta = 0$ in all scenarios are offered in

Table 3. Wind-load- electricity price scenarios and their probabilities

Solution	β	Total Cost (\$/h)	Variance (\$/h^2)
1	0	229228.2426	64539.9540
2	0.0526	229228.2425	64540.0588
3	0.1053	229228.2425	64540.0590
4	0.1579	229228.2425	64540.0594
5	0.2105	229228.2425	64540.0595
6	0.2632	229228.2425	64540.0595
7	0.3158	229228.2425	64540.0595
8	0.3684	229228.2425	64540.0595
9	0.4211	229228.2425	64540.0595
10	0.4737	229228.2425	64540.0596
11	0.5263	229228.2425	64540.0597
12	0.5789	229228.2425	64540.0598
13	0.6316	229228.2425	64540.0598
14	0.6842	229227.3363	64540.8891
15	0.7368	229227.3363	64540.8893
16	0.7895	229227.3363	64540.8893
17	0.8421	207479.5946	144419.7292
18	0.8947	207474.5862	144454.0335
19	0.9474	182099.2041	338231.2669
20	1	147455.3546	27074975.0013

Fig.11, also active and reactive power output of wind farm for $\beta = 0$ in all scenarios for this case are supposed in Fig.12.

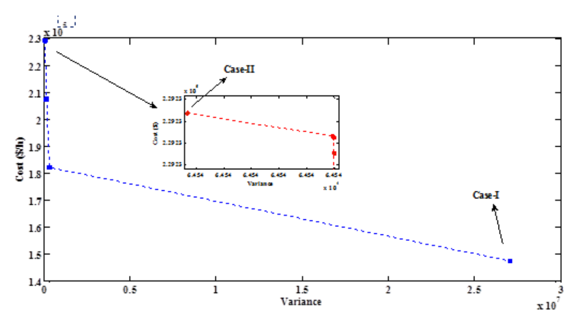

Fig. 9. The Pareto optimal front of Case-II.

C. Case-III: Considering risk via index

In order to solve the problem by -constraint method, Pareto optimal cost front of is depicted in Fig. 13. This Pareto front consists of 20 Pareto optimal solutions. The computational time in this case is 1243 sec. The values of the cost and for all 20 Pareto optimal solutions is presented in Table 4. Among these optimal

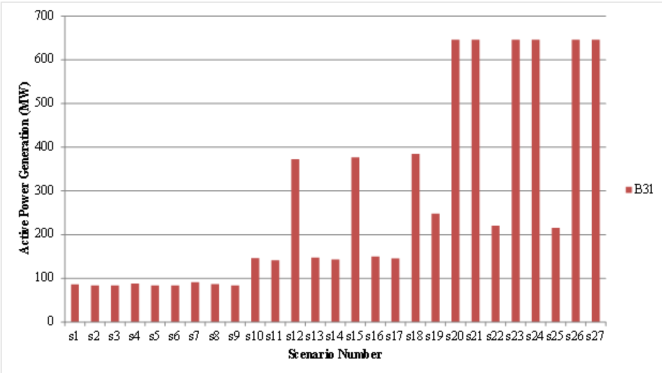

Fig. 10. Active power generation in slack bus (i.e. bus 31) in all scenarios (in MW) in Case-II.

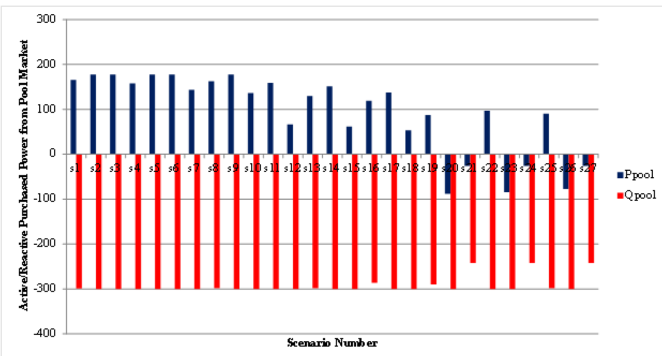

Fig. 11. Active and reactive purchased power from external network (located at bus 25) in all scenarios (in MW and MVAR) in Case-II.

solutions, for $\beta = 0$ the maximum cost, with the cost equal to 151035.0664 \$/h the is equal 159892.1459 \$/h2. For this solution, active power generation in slack bus for $\beta = 0$ in all scenarios is given in Fig.14. Additionally, active and reactive purchased power from pool market for $\beta = 0$ in all scenarios are given in Fig.15 and active and reactive power output of wind farm for $\beta = 0$ in all scenarios for this case are supposed in Fig.16.

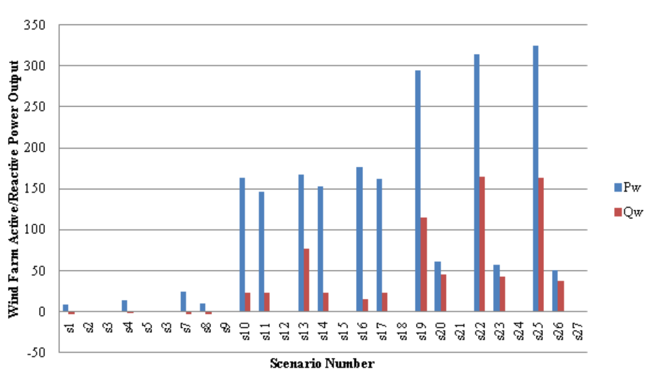

Fig. 12. Active and reactive power output of wind farm (located at bus 16) in all scenarios (in MW and MVAR) in Case-II.

Table 4. Pareto optimal solutions by considering risk management $CVaR$

Solution	β	Total Cost (\$/h)	CVAR (\$/h^2)
1	0	151035.0664	159892.1459
2	0.0526	150843.6086	159892.1459
3	0.1053	150652.1507	159892.1459
4	0.1579	150460.6929	159892.1459
5	0.2105	150269.235	159892.1459
6	0.2632	150077.7772	159892.1459
7	0.3158	149886.3193	159892.1459
8	0.3684	149694.8615	159936.5579
9	0.4211	149503.4037	159945.3872
10	0.4737	149311.9458	160041.1533
11	0.5263	149120.488	160148.0719
12	0.5789	148929.0301	160261.0179
13	0.6316	148737.5723	160390.4808
14	0.6842	148546.1145	160531.8501
15	0.7368	148354.6566	160675.4816
16	0.7895	148163.1988	160685.8033
17	0.8421	147971.7409	160734.4268
18	0.8947	147780.2831	160866.077
19	0.9474	147588.8252	161051.8215
20	1	147397.3674	161394.5729

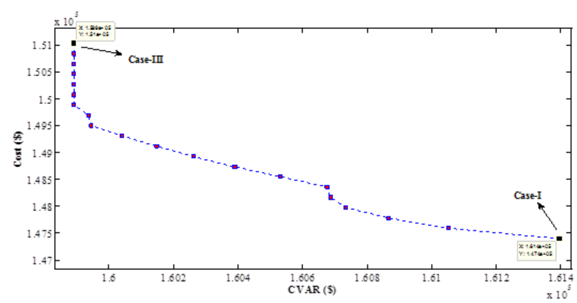

Fig. 13. The Pareto optimal cost front of Case-III.

D. Discussion on obtained results

In this paper active power generation of slack bus is considered as the "wait and see" variable and the PV buses active power generations are considered as the "here and now" variables. The power generation of slack bus in all cases has been presented in previous sections, but the PV buses power generations in all cases is depicted in Fig. 17. Also, the voltage magnitudes in all cases are presented in Table 5.

According to Fig.9 and Fig.13, if the risk coefficient (i.e. β)

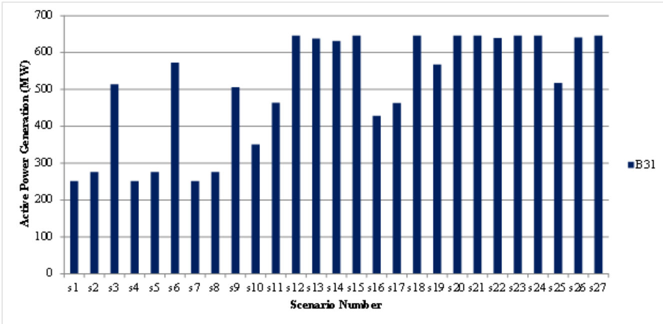

Fig. 14. Active power generation in slack bus (i.e. bus 31) in all scenarios (in MW) in Case-III.

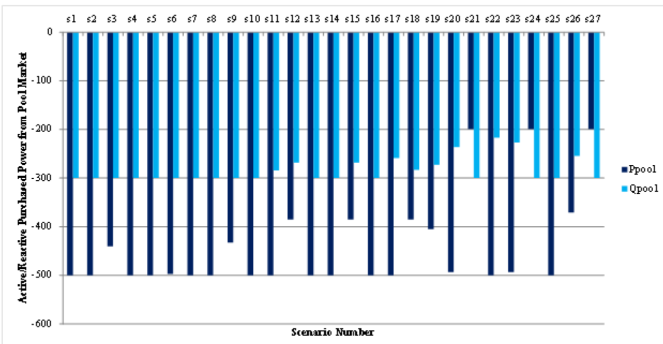

Fig. 15. Active and reactive purchased power from pool market (located at bus 25) in all scenarios (in MW and MVAR) in Case-III.

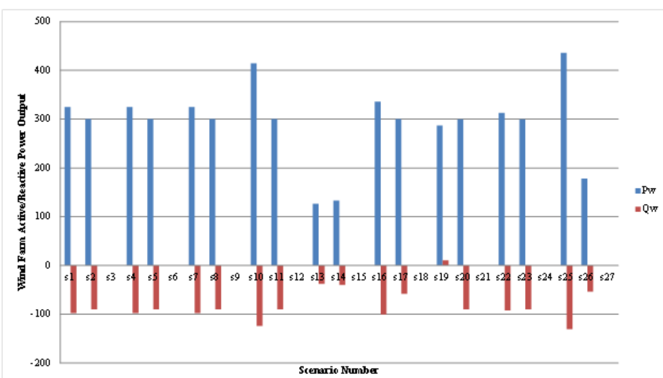

Fig. 16. Active and reactive power output of wind farm (located at bus 16) in all scenarios (in MW and MVAR) in Case-III.

being high, variance and CVaR will become low, with the low amount of risk, while the cost is higher than risk neutral case. Making a comparison between the results presented in Table 3 and Table 4 in $\beta = 0$ shows that the value of expected cost by considering is less than that of variance. Also, when β varies in the interval of [?, 1], the value of does not vary significantly, but the variance changes extremely. Therefore, index is more appropriate than the variance for taking risks and achieve lower cost.

Table 5. Voltage magnitudes of generator buses (pu) in all cases

Solution	β	Total Cost (\$/h)	CVAR (\$/h^2)
1	0	151035.0664	159892.1459
2	0.0526	150843.6086	159892.1459
3	0.1053	150652.1507	159892.1459
4	0.1579	150460.6929	159892.1459
5	0.2105	150269.235	159892.1459
6	0.2632	150077.7772	159892.1459
7	0.3158	149886.3193	159892.1459
8	0.3684	149694.8615	159936.5579
9	0.4211	149503.4037	159945.3872
10	0.4737	149311.9458	160041.1533
11	0.5263	149120.488	160148.0719
12	0.5789	148929.0301	160261.0179
13	0.6316	148737.5723	160390.4808
14	0.6842	148546.1145	160531.8501
15	0.7368	148354.6566	160675.4816
16	0.7895	148163.1988	160685.8033
17	0.8421	147971.7409	160734.4268
18	0.8947	147780.2831	160866.077
19	0.9474	147588.8252	161051.8215
20	1	147397.3674	161394.5729

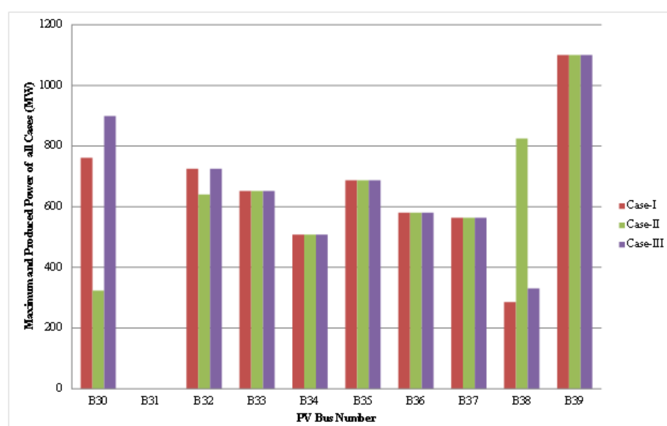

Fig. 17. Maximum and active power generation in PV buses (in MW) in all cases.

E. Value of Stochastic Solution

In order to evaluate the suitability of the proposed stochastic model for the OPF problem, value of stochastic solution (VSS) index [51] is calculated. This index highlight the impact of stochastic models in simulations.VSS is defined as follows:

$$VSS = EEV - SS \quad (36)$$

The VSS for both cases (case-II and case-III) is calculated using the results given in Tables 3 and 4. It is observed that in Case-II the VSS for total cost is 353.5811 \$/h. Also in Case-III this value is 513.6491 \$/h. These results imply that in stochastic model the cost values are more than deterministic case and because of this, the stochastic results are more realistic than the deterministic results.

5. CONCLUSIONS

In this paper stochastic version of OPF problem is studied via scenario-based modeling approach. Multiple sources of uncertainty such as wind power generation, power exchange with external network and load demand are considered. In order to manage the financial risk associated with these uncertainties, two indices i.e. variance and conditional value at risk (CVaR) are utilized. The proposed model is examined on the modified 39-bus New England test system. The obtained numerical results substantiate that Risk aversion leads to increase the total cost of energy supply. Hence, for a desired operation of electricity markets, each system operator should make a trade-off between the cost minimization and its associated risk. Also, the results confirm that CVaR is more appropriate index of risk reduction in the proposed risk constrained OPF model.

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