

Generation Expansion Planning in Hybrid Nuclear Systems and Renewable Energies Based on Uncertainty Modeling

Alireza Soheili Rostampour¹, Hamdi Abdi^{1,*} and Maryam Shahbazitabar¹

¹ Electrical Engineering Department, Engineering Faculty, Razi University, Kermanshah, Iran

*Corresponding author: Hamdiabdi@gmail.com

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In response to the escalating global demand for electricity, the development of new power generation facilities has become imperative. While nuclear and renewable power plants are two primary contenders in the modern energy landscape, each presents a unique set of benefits, challenges, and uncertainties that impact their efficiency and deployment. This study addresses Generation Expansion Planning (GEP) with the primary objectives of cost reduction, emissions minimization, and enhanced grid reliability through a lower Loss of Load Probability (LOLP). We proposed a hybrid system integrating nuclear and renewable sources to pave the way for a carbon-free grid, effectively mitigating the limitations inherent in each technology when operated independently. A key innovation of this research lies in the strategic integration of a Battery Energy Storage System (BESS) to directly address the intermittency of renewable sources and substantially improve grid reliability. To accurately model the real-world uncertainties affecting this hybrid system, including energy demand, fuel prices, wind speed, solar irradiation, and plant depreciation rates, we employ Probability Density Functions (PDFs) and Beta distributions. The stochastic behavior of these variables is then simulated using a Markov Chain Monte Carlo (MCMC) method for robust forecasting. The model is subsequently optimized using a Stochastic Dynamic Optimization algorithm. The efficacy of our introduced BESS is demonstrated through a significant improvement in a critical reliability metric. Our simulation results show that the cumulative LOLP index over a 13-year planning horizon was reduced from 2.85 to 2.33. This marked improvement was achieved with an initial BESS capacity of only 20 MW in the first year, expanded at a modest annual growth rate of 10%, highlighting the practical efficiency and scalability of our presented solution.

Keywords: Generation Expansion Planning, SMART Reactors, Renewable Resources, Uncertainty Analysis, Markov Chain Monte Carlo, Stochastic Dynamic Programming, Battery Energy Storage Systems

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Nomenclature

μ_I, μ_f^n, μ_w	Mean value	$E[\max(0, D_{dep}(C_{current}+X))]$	The function returning positive values
$\sigma_I^2, \sigma_f^{n2}, \sigma_w^2$	The variance of the variable x_I, n_{th}, x_w	D	The load demand
α and β	The constant parameters of the Beta density function	$C_{current}$	The current capacity
Γ	The Gamma function	dep	The mean of the dataset
$f(y)$	The density function of depreciating	$LOLP(t)$	The Loss of Load Probability value at time (year)
$F_{(x)}$	The objective function in \$.	N	The total number of generating units or samples
C_{inv}	The investment cost in \$/MW.	I	The indicator function; it equals 1 if the condition inside the parentheses is "true," and 0 otherwise.
X	The decision variable, i.e., new investment, in MW.	$D_{t,i}$	The load demand for unit *i* at time *t*.
C_{pen}	The penalty cost for load shortfall in \$/MW.	C_t	The total system capacity at time *t*
		ε_i	Confidence margin for the reserve of unit *i*
		$C_{(t+1)}$	The capacity for the next year

C_T	The capacity in the current year
I_t	The investment in the current year in MW
δ	The annual depreciation rate
$YearCost_{(t)}$	The annualized cost in \$.
X_t	The number of units ordered or invested in during year t^*
$E[S_t]$	The expected shortfall in year t^* , representing the average negative inventory throughout the year
$(1+r)^{t-1}$	The discount factor, which converts costs from year t^* to their present value
N	The number of life cycles until reaching 80% of the initial capacity
DOD	The battery's Depth of Discharge
m_0	The battery's Depth of Discharge
m_1 and m_2	Exponential coefficients, whose values fluctuate based on information provided by the battery manufacturer
A_b	The capacity reduction per cycle in (MWh/cycle)
E_b^{lni}	The initial battery capacity in (MWh)
CF_t	The net cash flow at year t
R	The discount rate
N	The project lifetime

1. Introduction

To meet the rising demand for electricity, constructing new power plants has become essential, and optimizing their expansion at minimal cost is a key priority; however, the environmental impact of pollutant emissions from these facilities remains a significant concern. GEP for hybrid nuclear-renewable power plants offers a modern solution for achieving a sustainable energy supply, reducing greenhouse gas emissions, and enhancing energy security by combining the stable base-load capacity of nuclear energy with the variable output of renewable sources like solar and wind. This approach begins with forecasting future electricity demand to determine the necessary capacity from each source, supported by mathematical models and advanced simulations that help identify the optimal mix for cost-effective, efficient, and environmentally responsible development. A major advantage of hybrid systems is their ability to improve grid flexibility, with nuclear power ensuring consistent supply and renewables meeting peak-load demands during periods of high consumption.

From an economic perspective, hybrid power plants can create a more favorable cost structure for countries by reducing fuel expenses and dependence on fossil resources. Although nuclear power plants require high initial investment, they have low operational costs in the long term and can reduce overall electricity generation costs when combined with renewable energy sources. Furthermore, the development of these power plants can generate new economic opportunities, create jobs in clean technology sectors, and promote growth in industries related to renewable energies.

From an environmental perspective, the use of hybrid nuclear-renewable power plants has numerous positive impacts. Nuclear power plants produce no direct greenhouse gas emissions, while renewable energy sources further reduce the reliance on fossil fuels. This combination helps decrease air pollution, mitigate climate

change, and reduce dependence on polluting energy sources. However, managing radioactive waste from nuclear power plants remains a significant environmental challenge, requiring advanced techniques for storage and recycling of radioactive materials, which will be discussed in the following section. However, the uncertainty in the future supply of fossil resources presents another concern for central planners and investors. Despite all the environmental advantages of Renewable Energy Sources (RES) in power systems, their widespread integration can adversely affect electricity grids due to their non-dispatchable and variable output. The challenges posed by RES include: limited available transmission line capacity for their deployment, new requirements for frequency and voltage regulation, necessary installations and grid connections, as well as disruptions to the operation of conventional units. These disruptions manifest as an increased number of start-ups and shutdowns for thermal power plants and their reduced efficiency. In general, hybrid energy refers to a combination of renewable and thermal energy sources. In particular, this integration offers significant advantages when it includes nuclear power units. Integrating renewable energy sources into the power grid is challenging due to the uncertainty of their output caused by weather conditions. A distinct category of approaches exists to incorporate uncertainty into models. Capacity value was introduced to address this uncertainty by considering the worst-case generation output scenario, which represents the minimum power that can be produced.

A two-stage robust optimization method is used to address the wind energy uncertainty in the GEP problem. The first step models the long-term impacts of wind uncertainty using the annual variation of wind capacity factor, while the second step addresses short-term uncertainties by applying the autoregressive moving average (ARMA) technique.

Stochastic programming is employed to address inherent uncertain parameters in planning, providing an optimal solution for all scenarios. The scenarios are designed to describe the actual range of all uncertain planning parameters. The computational burden can increase significantly depending on the level of detail used to represent the stochastic process. The most common practical tool for handling uncertainties is the Monte Carlo method. The uncertain parameters must first be described by a Probability Density Function. The simple Monte Carlo method using a single PDF is not accurate for calculating time-dependent, continuous uncertainty. Consequently, data classification methods are necessary to represent the probabilistic correlation between different uncertainty sources based on long-term historical data, thereby generating time-dependent values. The Markov Chain Monte Carlo method was introduced to forecast the stochastic behavior of uncertainty sources. This method can be used to generate samples considering the temporal persistence of values, long-term trends, and the fitted PDF of the uncertainty sources. Accordingly, the Metropolis-coupled Markov Chain Monte Carlo method was introduced to simulate the stochastic behavior of uncertainty sources with a multimodal PDF. The growing energy and environmental crises have spurred the widespread adoption of diverse renewable energy sources, such as wind turbines and photovoltaics (PVs), to meet consumer energy demands. Integrating a significant amount of renewable energy into the power system leads to a substantial increase in power flow uncertainty, consequently profoundly impacting voltage stability and frequency control within the power system. While Battery Energy Storage Systems (BESS) are essential flexible resources in power systems, their ability to shift electrical energy across both space and time, coupled with their bidirectional power support capabilities, plays a crucial role. This includes enhancing renewable energy integration, smoothing renewable power fluctuations, optimizing power flow, improving voltage stability, and addressing challenges such as insufficient peak-shaving capacity and emergency response. BESS can effectively increase the flexibility and overall reliability of the power grid.

However, the over-implementation of BESS can lead to the under-utilization of resources and negatively impact the overall economic efficiency of the power grid. Simultaneously, prolonged periods of inactivity for battery components result in reduced operational efficiency. Furthermore, when the capacity configuration of the BESS is insufficient, leveraging the flexible benefits of energy storage resources becomes challenging. An inadequate BESS capacity not only hinders the effective absorption of surplus power generated by wind turbines and photovoltaics but also restricts the exploration of additional application scenarios to enhance profitability.

A useful summary of the ETAP software, including its application in power system planning, can be found in [19].

Ref. [20] focuses on power infrastructure development in India, where most plants rely on traditional energy sources. Using GEP, it analyzes system performance over 6- and 14-year horizons, particularly the impact of adding solar power plants with or without storage. A four-level hierarchy is applied to evaluate investment strategies, replacement of high-emission plants, and emission penalties. Sensitivity analyses consider solar penetration levels, costs, capacity additions, and reliability (EENS, FOR%). Results show that including solar plants with storage offers promising prospects for future power system planning while highlighting the complexity of policy and decision-making in GEP.

In [21], the integration of renewable energy into power systems was addressed, highlighting the challenges of variable resource availability. It proposes a stochastic programming approach to optimize GEP, explicitly accounting for generator capacity uncertainty. Flexibility is incorporated through real options, modeled with decision rules that combine conditional-go and finite adaptability principles. The method provides practical guidelines for decision makers, with threshold values for exercising options. A case study in the Midwest United States illustrates how the approach quantifies the value of flexibility and demonstrates its effectiveness in power system planning.

The proposed method offers several distinct advantages over conventional approaches:

- Overcoming statistical simplification: Rather than assuming a normal distribution for all variables, this method applies statistical distributions aligned with the physical nature of each parameter (normal for demand, beta for solar, truncated normal for wind). This significantly enhances the accuracy of modeling uncertainties.
- Intelligent MCMC sampling: Inefficient random sampling is replaced with the MCMC method combined with a distance-based weighting technique. This improves computational efficiency and substantially increases the accuracy of estimates.
- Dynamic optimization strategy: Instead of delivering a static, fixed solution, the method employs a year-by-year optimization framework. This dynamic approach enhances decision-making flexibility and enables adaptation to evolving market conditions.

Finally, all calculation steps in this method are fully transparent and traceable, ensuring reliability and reproducibility.

Based on a review of the literature and related studies, the most significant innovations, and goals of this research are as follows:

- Address the GEP with primary objectives of reducing costs, minimizing emissions, and enhancing grid reliability by lowering the LOLP.
- Consider the use of a BESS to mitigate the intermittency of renewable sources and substantially improve grid reliability.

- Model the uncertainty set of demand, fuel prices, wind speed, solar irradiation, and plant depreciation rates using a MCMC approach.
- Apply a stochastic dynamic optimization algorithm to address the optimization problem.
- Increase network reliability by accounting for the impacts of various uncertainties.

2. Methodology

This section is dedicated to describing the methodology of the present study, and the research methodological framework is presented step by step in full detail. Considering the interdisciplinary nature of this thesis, which is a combination of energy engineering, systems planning, and uncertainty analysis, the research methodology has been designed in a way that can respond to all the research objectives. First, a general introduction to the research method is given, and then the implementation stages of the research are described in detail. Next, data collection methods, sample selection criteria, and tools used for data analysis are presented. The main part of this section is dedicated to describing the proposed model and the Monte Carlo method for modeling uncertainties, which form the core of this thesis. Finally, the model validation methods and parameter sensitivity analysis are discussed, and the ethical considerations of the thesis are also stated. The necessary scientific basis for the analyses and results will be presented in the following subsections. Therefore, this study presents two scenarios with and without energy storage devices to compare these two scenarios and also the effects of this generation of batteries on network reliability.

2.1. Uncertainty

Electricity demand has always been characterized by inherent uncertainties due to the dynamic nature of consumption across industrial, commercial, and residential sectors. These uncertainties become more complex in hybrid systems that combine nuclear power plants and renewable energy sources, as such systems must achieve stable balance while simultaneously dealing with fluctuations in variable generation (such as solar and wind power) and unpredictable changes in consumption. This section examines the root causes, impacts, and management strategies for load demand uncertainties. Factors such as seasonal consumption patterns, economic growth, demographic changes, and even social behaviors can lead to significant deviations between forecasts and actual outcomes. To account for electricity demand uncertainty, the total demand from the previous year and the PDF of load demand for each year are considered. The uncertainty of the load demand is modeled by its PDF, along with the demand from the previous year, as follows [22].

$$f(x_t) = p(X = x_t | \mu_t, \sigma_t^2) = \frac{1}{\sqrt{2\pi\sigma_t^2}} \exp \left[-\frac{(x_t - \mu_t)^2}{2\sigma_t^2} \right] \mu_t, \sigma_t \in (0, \infty) \quad (1)$$

Where μ_t is the mean value and σ_t^2 is the variance of the variable x_t . In the modern era, one of the most significant challenges facing hybrid power systems is the uncertainty in the price of energy carriers. This uncertainty, stemming from numerous political, economic, and environmental factors, directly impacts the economics of operation and the development planning of nuclear-renewable hybrid systems.

The price of energy carriers is another factor that can significantly influence the selection of candidate units for development. Fluctuations in fuel prices over the study period can impact the development trajectory. Consequently, a decrease in the fuel cost of a generation unit encourages the optimization process to commission that unit due to the resulting reduction in the project's final development cost, and vice versa. The uncertainty in energy price is also formulated using a standard PDF.

(2)

$$f(x_f^n) = p(X = x_f^n | \mu_f^n, \sigma_f^{n2}) = \frac{1}{\sqrt{2\pi\sigma_f^{n2}}} \exp\left[-\frac{(x_f^n - \mu_f^n)^2}{2\sigma_f^{n2}}\right] n \in \{N\}, \mu_f, \sigma_f \in (0, \infty)$$

Where μ_f^n is the mean and σ_f^{n2} is the variance of the n_{th} value of the random variable x_f . Each fuel type has its own specific μ_f and σ_f^2 .

Wind energy, as a cornerstone of renewable energy systems, faces inherent challenges of uncertainty in its power generation. These uncertainties, which stem primarily from the innate variability of wind resources, have a direct impact on grid stability and the operational economics of hybrid systems. These power plants introduce uncertainty when supplying electricity to the grid because wind speed and direction are variable. Fluctuations in wind speed and direction alter the pitch angle, the angle of attack, and the rotational speed of the wind turbine connected to the generator. Consequently, the shaft speed changes, leading to variations in the active power injected by the wind farm due to changing wind speeds. Therefore, the uncertainty in a wind unit's output is due to changes in wind speed and direction. The uncertainty associated with wind turbine output power can be modeled using a Normal PDF, which is referred to as model uncertainty.

(3)

$$f(x_w) = p(X = x_w | \mu_w, \sigma_w^2) = \frac{1}{\sqrt{2\pi\sigma_w^2}} \exp\left[-\frac{(x_w - \mu_w)^2}{2\sigma_w^2}\right] \mu_w, \sigma_w \in (0, \infty)$$

Where μ_w is the mean and σ_w^2 is the variance of the variable x_w .

Solar energy, as one of the most promising renewable resources, faces significant challenges related to uncertainty in power generation. These uncertainties, which arise primarily from the variable nature of solar irradiance, have a direct impact on grid stability and the operational planning of power systems. Solar power plants face uncertainty when delivering electricity to the grid because solar irradiance and weather conditions are subject to frequent fluctuations.

Solar cells are engineered within a solar module to directly convert solar radiation into electrical energy, and changes in irradiance and ambient temperature can alter the output power of a PV unit. Therefore, the uncertainty in the power generation of a PV unit is due to variations in solar irradiance at the panel's surface. The uncertainty in PV power plant generation is formulated using a Beta PDF.

$$f(x_p) = p(X = x_p | \alpha, \beta) = \begin{cases} \frac{\Gamma(\alpha + \beta) x_p^{\alpha-1} (1 - x_p)^{\beta-1}}{\Gamma(\alpha) \Gamma(\beta)} & \forall 0 \leq x_p \leq 1 \\ 0 & \text{otherwise} \end{cases} \quad (4)$$

where α and β are the constant parameters of the Beta density function. They must be selected as desired values based on the specific PV power plant under consideration, and Γ is the Gamma function.

The depreciation rate of a power plant is another factor that may contribute to the plant reaching the end of its operational life and subsequently being decommissioned. Since the depreciation behavior of a generating unit depends on its technology, operating conditions, and service age, it can be represented by a probability density function over the study horizon. Accordingly, the annual depreciation rate is modeled using the following formulation:

$$f(y) = \frac{(T - y)\beta^2}{(T - \beta y)T[(1 - \beta)\ln(1 - \beta) + \beta]} \quad (5)$$

The parameter β determines the shape and temporal distribution of the depreciation profile. For $0 < \beta < 1$, the depreciation function decreases with increasing plant age y , indicating relatively higher

annual depreciation during the earlier years of the plant lifetime followed by a gradual decline in subsequent years. As β approaches unity, the depreciation profile becomes more gradual and less front-loaded, with a larger proportion of the depreciation distributed toward the later stages of the useful lifetime. In this study, $\beta=0.9$ is assumed for the SMART plant to represent a relatively gradual depreciation profile, whereas $\beta=0.6$ is assumed for the conventional diesel plant to represent a comparatively more front-loaded depreciation profile. These values are adopted as modeling assumptions in the absence of technology-specific historical depreciation data. The depreciation function is evaluated for each year of the planning horizon and subsequently normalized by the sum of its annual values to obtain the depreciation weights used in the generation expansion planning model.

2.2. Markov Chain Monte Carlo

The MCMC method is based on two functions: the prior Probability Density Function and the likelihood functions. The PDF is calculated by performing curve fitting on the available historical data histogram. The likelihood function represents the probability of transitioning from one generated value to another, determined based on the concept of the Markov chain model. Consequently, the MCMC method can improve the forecasting of the stochastic behavior of uncertainty sources.

The uncertainties mentioned in the previous section, along with their corresponding PDFs, need to be sampled to be incorporated into the proposed algorithm. This sampling must be performed accurately because the results from this stage must be analyzed by the optimization algorithm in the subsequent section. For this purpose, the Markov Chain Monte Carlo method is used to sample the density functions. The proposed PDF for the Markov chain is considered to be a standard PDF, expressed as follows:

$$\text{Proposed PDF} = f(X = x | IS, \sigma^2) \quad (6)$$

Where the initial proposed state (IS) was selected, and σ^2 was set equal to 1. Subsequently, sampling was performed from all functions, and all samples were fed into the optimization section. In the first stage, the density functions were defined as a function in MATLAB, and then the acceptance constraint was verified using the following relation [1]:

$$\omega = \min\left[1, \frac{f(X = x^{\text{next sample}})}{f(X = x^{SI})}\right] \quad (7)$$

$$\Omega = \text{uniform}(0,1) \quad (8)$$

In this approach, the candidate distribution function is aligned with the uncertainty profile. To determine the acceptance of each point, the ratio between the current and previous sample probabilities—denoted as ω —is evaluated against a threshold of 1. Subsequently, a stochastic value Ω is generated via a uniform distribution [0, 1]. A sampling point is integrated into the dataset only if ω exceeds or equals Ω ; otherwise, it is discarded without further consideration. This iterative procedure must be executed extensively to ensure the convergence and reliability of the final results. Furthermore, a statistical tolerance range (L, U) is defined to encompass at least a fraction γ of the total population, maintained at a specific confidence level β . These parameters are pre-defined by the researcher based on the necessary precision. To optimize efficiency, Wilks' methodology is employed, allowing for a significant reduction in the required sample size—often from several thousand down to roughly 100—regardless of how many uncertain variables are present in the underlying model [26].

2.3. Stochastic Dynamic Optimization

There are two types of algorithms for solving the GEP problem: static algorithms and dynamic algorithms. A static algorithm is a

simpler approach for development, but its accuracy is significantly lower compared to a dynamic algorithm because it does not allow for the addition or removal of generation capacity between the base year and the target year. Consequently, the accuracy of the plan decreases as it approaches the final years of the study.

This paper optimizes the power plant development process using a stochastic-dynamic approach. The problem assumptions include unit characteristics such as fuel cost, unit emission rates, fuel heat rate, operation and maintenance information, etc. and expansion constraints such as spinning reserve margin, system reliability index, and greenhouse gas emission limits. The objective is to calculate the optimal plant capacity among candidate units to minimize the objective function while satisfying all problem constraints.

In power system expansion planning, decision-makers often face trade-offs among multiple conflicting objectives, where optimizing one index inevitably degrades others. To address this, the total cost is designated as the primary objective function in this study, while other performance criteria are modeled as system constraints. Under this formulation, the cost is treated as a continuous variable to be minimized, whereas the number of newly planned power plants is a discrete decision variable determined directly by the optimization algorithm. The objective function is defined as the sum of two components: the investment cost associated with the expansion decision and the expected penalty for supply-demand mismatch. The term $C_{inv}(X)$ represents the capital investment required to implement the additional capacity X , while the second term captures the economic impact of any shortage that may arise after expansion. In this formulation, $D_{dep}(C_{current}+X)$ describes the demand-related imbalance as a function of the total available capacity, which is the combination of the existing capacity $C_{current}$ and the newly added capacity X . The operator $\max(0, \cdot)$ ensures that only positive values of shortage contribute to the penalty, meaning that no cost is imposed when the available capacity is sufficient to satisfy demand. By taking the expectation of this term, the model accounts for uncertainty in demand conditions and evaluates the average shortage risk across possible realizations. As a result, the objective function provides a balanced planning framework that minimizes both the cost of expansion and the expected consequence of insufficient capacity.

$$\min F(x) = C_{inv}(X) + C_{pen} \times E[\max(0, D_{dep}(C_{current}+X))] \quad (9)$$

The proposed generation expansion planning framework integrates power system adequacy, dynamic capacity evolution, economic performance, and demand satisfaction into a unified optimization model. The mathematical formulation is based on well-established concepts of probabilistic reliability assessment and power system planning, while several performance indices are specifically formulated to evaluate the proposed planning strategy.

Power system adequacy is evaluated using the Loss of Load Probability (LOLP), one of the most widely adopted reliability indices in generation adequacy assessment. LOLP represents the probability that the system demand exceeds the available generation capacity and is extensively used in probabilistic power system reliability studies [27, 28]. In this study, the probability is estimated using a Monte Carlo sampling approach, where the expected probability is approximated by the sample average of an indicator function as:

$$LOLP(t) = \frac{1}{N} \sum_{i=1}^N I(D_{t,i} > C_i) \quad (10)$$

Where N denotes the total number of Monte Carlo samples, $D_{t,i}$ is the sampled electricity demand at year t , and $I(\cdot)$ is the indicator function that equals one whenever the sampled demand exceeds the

available generation capacity and zero otherwise. The available generation capacity in each sample is modeled as:

$$C_i = C_t(1 + \varepsilon_i) \quad (11)$$

Where C_t denotes the installed generation capacity in year t , and ε_i the stochastic uncertainty associated with the available generation capacity.

To represent the long-term evolution of the generation system, the available capacity is updated annually by considering both newly installed capacity and the gradual degradation of existing assets. Accordingly, the available generation capacity for the subsequent planning period is calculated as:

$$C_{t+1} = (C_t + I_t)(1 - \delta) \quad (12)$$

Where I_t denotes the newly installed generation capacity during year t , while δ represents the annual depreciation coefficient of the generation fleet. This recursive relationship is adopted to capture the dynamic evolution of installed capacity throughout the planning horizon.

The economic performance of the expansion plan is evaluated through the discounted annual system cost. The objective function considers both investment expenditures and the expected economic penalty associated with supply shortages. Therefore, the annual system cost is expressed as:

$$YearCost_t = \frac{X_t C_{inv} + E[S_t] C_{pen}}{(1+r)^{t-1}} \quad (13)$$

Where X_t denotes the installed capacity expansion during year t , C_{inv} is the unit investment cost, $E[S_t]$ represents the expected energy not supplied (or expected supply shortage), C_{pen} is the unit interruption penalty cost, and r is the annual discount rate. The discount factor accounts for the time value of money throughout the planning horizon.

In addition to the economic objective, an index is introduced to quantify the capability of the proposed expansion strategy in satisfying the system demand. Unlike conventional energy conversion efficiency, this indicator evaluates the expected proportion of electricity demand successfully supplied during the planning horizon and is therefore referred to as the Demand Supply Efficiency Index (DSEI) in this study. The index is defined as:

$$DSEI = \frac{\sum_t E[D_t](1 - LOLP_t)}{\sum_t E[D_t]} \quad (14)$$

Where $E[D_t]$ denotes the expected electricity demand during year t . Since $1 - LOLP_t$ represents the probability that the system is capable of meeting the demand, the numerator corresponds to the expected supplied demand, whereas the denominator represents the total expected demand over the planning horizon. Consequently, the proposed index provides a normalized measure of the effectiveness of the generation expansion strategy in satisfying electricity demand under uncertainty.

The above mathematical formulation simultaneously incorporates reliability assessment, dynamic capacity evolution, economic evaluation, and demand satisfaction into the proposed optimization framework, thereby enabling a comprehensive assessment of long-term generation expansion strategies under uncertain operating conditions.

2.4. BESS

The degradation of lithium-ion batteries is generally attributed to two major mechanisms, namely cycle aging and calendar aging. Cycle

aging is mainly influenced by the charge/discharge cycling conditions, particularly the depth of discharge (DOD) and the accumulated number of charge/discharge cycles, whereas calendar aging is primarily associated with storage time and environmental conditions such as temperature, irrespective of battery cycling.

In this study, battery degradation is evaluated based on the cycle-aging mechanism, since the objective is to investigate the influence of battery energy storage systems (BESS) on generation expansion planning and system reliability. The relationship between the cycle life and the depth of discharge is represented using an empirical DOD-dependent degradation model, which belongs to the family of nonlinear curve-fitting models widely adopted for lithium-ion battery lifetime estimation:

$$N = m_0 \times DOD^{-m_1} \times e^{m_2(1-DOD)} \quad (15)$$

The charge and discharge behavior corresponding to a specific DOD value uniformly affects BESS degradation until the maximum cycle count is reached. Furthermore, the impact of charge and discharge cycles on degradation is completely independent of previous cycles.

The battery degradation is modeled using a linear approximation based on the standard end-of-life (EOL) criterion. Assuming the battery reaches its EOL when the capacity fades to 80% of its initial value E_b^{ini} after N cycles, the average capacity reduction per cycle, based on the above assumptions, the capacity reduction resulting from the charge/discharge process can be estimated in relation to the BESS lifespan using the following equation:

$$\Lambda_b = \frac{0.2 \times E_b^{ini}}{N} \quad (16)$$

This formulation provides a computationally efficient representation of the aging process, suitable for long-term planning and optimization studies. In modeling the BESS for this research, the objective is to demonstrate the impact of this generation of energy storage systems on reliability. Consequently, only the fundamental cycle degradation formulas have been used. To optimize this generation, a fully linearized model suitable for integration into the optimization framework must be implemented.

2.5. Financial Evaluation Framework

The economic feasibility of the power generation expansion project was evaluated using a comprehensive discounted cash flow (DCF) analysis. The primary metric for investment attractiveness is the net present value (NPV) and the subsequent return on investment (ROI) derived from it, calculated over the project's assumed 13-year lifetime. This approach incorporates the time value of money, providing a more accurate financial assessment than simple payback period calculations [29].

The fundamental financial evaluation formulas applied are as follows:

Net present value (NPV):

$$NPV = \sum_{t=0}^n \frac{CF_t}{(1+r)^t} \quad (17)$$

ROI based on NPV:

$$ROI = \left(\frac{NPV}{|PV_{Investment}|} \right) \times 100\% \quad (18)$$

Payback period:

The payback period is defined as the minimum time T required for the cumulative discounted cash flow to become non-negative [29]:

$$\text{Payback period} = \min T, \text{ such that } \sum_{t=0}^n \frac{CF_t}{(1+r)^t} \geq 0 \quad (19)$$

3. Data analysis

This section presents a comprehensive analysis of the data generated from the uncertainty modeling and the stochastic-dynamic optimization simulations. The primary objectives are to quantify the impact of key uncertainties on the GEP outcomes and to perform a comparative evaluation of the scenarios with and without BESS. Fig. 1 illustrates the flowchart of the method used for GEP. All simulations and optimizations were conducted in the MATLAB environment. The initial input data, including the base load profile and the parameters of the generation units, were extracted from reliable sources. Specific PDFs were used, as shown in Table 1 and Table 2.

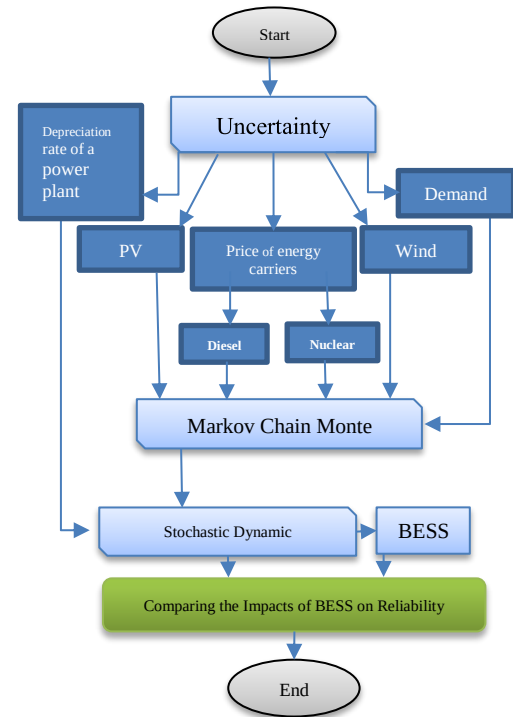


Fig. 1. Flowchart of the research

Table 1. The parameters of the PDF of the uncertainties.

Uncertainty	PDF
The depreciation rate of the unit	$0/6 \leq \beta \leq 0/9$
Wind unit output	$f(x_w) = p(X = x_w 10, 2)$
Solar PV unit output	$f(x_p) = p(X = x_p 2, 4)$
Price of nuclear fuel	$f(x_f^n) = p(X = x_f^n \frac{64}{4}, 25)$
Price of gasoline	$f(x_f^g) = p(X = x_f^g \frac{6}{6}, 1/6)$

Table 2. System Specifications.

Parameter	SMART	Diesel	PV	Wind
Minimum operating level (MWe)	20	50	7	8
Maximum generating capacity (MWe)	90/9	55	10	12
Domestic fuel costs (¢/106)	64/43	6/6	0	0

kcal)				
Maintenance class size (MW)	90.9	55	10	100

Some limitations of this research are:

Planning is for a 13-year period

Discrete annual modeling (not continuous)

No dependence between different years in modeling

Assumption of linear growth of BESS power

Constant network parameters.

4. Results

The long-term load demand profile was modeled to account for forecast uncertainty. Annual load growth was represented by a normal distribution with mean 100 MW and standard deviation 30 MW, constant over the 13-year period, and independent of year-to-year variation. The base year-1 demand was 500 MW, with growth assumed independent across years (no temporal correlation)..

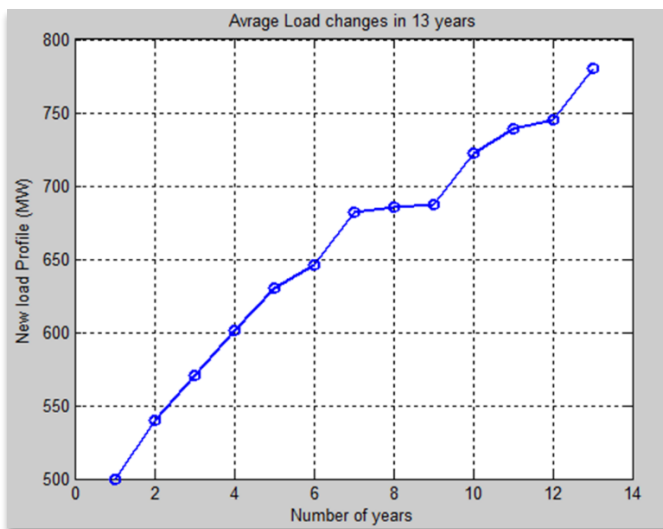


Fig. 2. The average load changes

Fig. 2. illustrates the trend of load demand changes over the 13-year horizon under various uncertainty scenarios. As evident from Fig. 2, the load demand follows a non-linear, upward trajectory, increasing from 500 MW in the first year to approximately 800 MW in the final year. This growth of nearly 60% clearly justifies the necessity for planning new generation capacity expansion.

Another major source of uncertainty in energy system expansion optimization is fuel price volatility. In this research, the prices of energy carriers for nuclear and diesel power plants as two foundational technologies were modeled using normal distributions.

- **Nuclear Fuel:** Characterized by a high standard deviation (25), this energy carrier is subject to very significant uncertainty. The price range across samples primarily fluctuates between approximately \$10 and \$120.

- **Distribution:** Normal with a mean of \$64.4 and a standard deviation of \$25.

- **Diesel Fuel:** Although it has a lower mean price, its coefficient of variation ($CV = \sigma/\mu$) is about 0.24, indicating considerable relative volatility around the mean.

- **Distribution:** Normal with a mean of \$6.6 and a standard deviation of \$1.6.

- **Optimal Energy Prices:** The analysis suggests optimal price

ranges of \$60-70 for nuclear energy and \$6-7 for diesel.

The results demonstrate that fuel price uncertainty has the most substantial impact on the total system cost.

One of the key sources of uncertainty in modern energy systems is the variable generation of wind farms. In this study, the wind farm output power is modeled using a truncated normal distribution. The choice of this distribution is motivated by the fact that, in many regions, wind speed patterns closely follow the Weibull or Rayleigh distributions. However, under specific conditions—such as when the average wind speed is high and variability is low—the normal distribution provides an acceptable approximation. Moreover, the truncated normal distribution offers greater analytical tractability than the Weibull distribution and is more convenient for applications such as MCs, reliability analysis, integral calculations, and random number generation. This distribution was chosen because it more realistically reflects the behavior of wind farms, which have a minimum (zero) and a maximum (rated capacity).

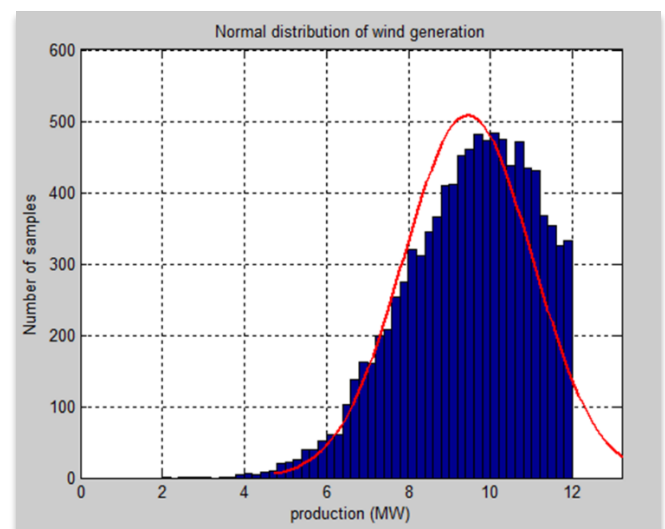


Fig. 3. Normal distribution of wind generation

As can be seen in Fig. 3, the selected model has successfully simulated the behavior of a real wind power plant. The presence of a left tail (slight skewness) in Fig. 3 results from truncating the distribution within the [0, 12] MW range, which aligns perfectly with the physical reality of plant capacity limits.

The mean output of 9.98 MW indicates that, on average, the plant operates at approximately 83% of its rated capacity. This figure is consistent with the capacity factors reported for many wind power plants worldwide. Given the intermittent and weather-dependent nature of solar power generation, modeling its uncertainty is of paramount importance. For this purpose, the Beta distribution was employed. This distribution is particularly suitable for modeling normalized variables, such as the output power of solar panels (relative to their maximum rated power), due to its flexibility in shape and its bounded support within the [0, 1] interval.

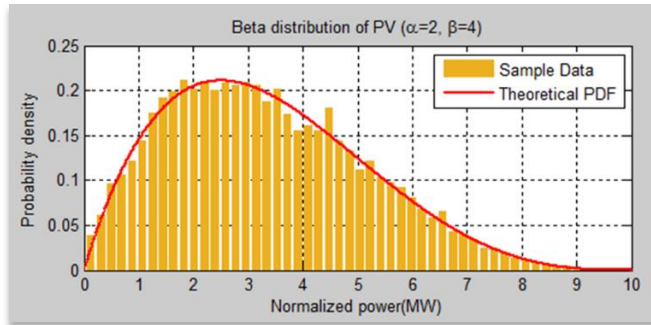


Fig. 4. Beta distribution of PV

As observed in Fig. 4, the PV output power distribution exhibits right-skewness (positive skew). This indicates that the probability of observing values below the mean is higher than that of values above it. This result is entirely consistent with the real-world behavior of solar power plants, as their output significantly decreases during many hours of the day (such as early morning or evening) and on cloudy days, only approaching its maximum capacity for limited hours on sunny days. The mean generation value of 3.33 MW indicates that, on average, this power plant operates at only about 33.3% of its rated capacity. This figure represents the system's capacity factor, which is typical for solar power plants.

One of the key parameters influencing the calculation of power plant life-cycle costs and optimal capacity expansion planning is the equipment depreciation rate. In this study, the depreciation profiles of two power generation technologies, namely the SMART reactor and conventional diesel power plant, are modeled using the depreciation function presented in Equation (5), which defines the temporal distribution of depreciation over the useful lifetime of a generating unit. The parameter β acts as a shape parameter that determines how depreciation is distributed over the plant lifetime. For $0 < \beta < 1$, the annual depreciation profile decreases as the plant age y increases, indicating relatively higher depreciation during the earlier years followed by a gradual decline over the remaining useful lifetime. A value of $\beta=0.9$ is assumed for the SMART plant. This relatively high value produces a gradual and less front-loaded depreciation profile, meaning that the annual depreciation is distributed more evenly over the plant lifetime compared with a lower β value. The selection of $\beta=0.9$ in this study is therefore intended to represent a relatively gradual depreciation pattern and should be regarded as a modeling assumption rather than a technology-specific empirical parameter. For the conventional diesel power plant, $\beta=0.6$ is assumed. Compared with $\beta=0.9$, this lower value results in a more front-loaded depreciation profile, with a relatively larger share of the depreciation assigned to the earlier years of the plant lifetime. The value is adopted as a modeling assumption to represent a depreciation profile different from that of the SMART plant.

The resulting depreciation profiles affect the annual depreciated capital cost assigned to each generating technology in the optimization model. Consequently, differences in the assumed depreciation profiles influence the timing and economic attractiveness of investments over the planning horizon. In particular, the less front-loaded depreciation profile adopted for the SMART plant can result in a different temporal distribution of capital depreciation compared with the diesel alternative. Therefore, the economic preference for one technology over another is determined jointly by the depreciation profile and the other technical and economic characteristics incorporated into the optimization model.

Table 3. Cumulative Depreciation Share

Power Plant Type	Cumulative Depreciation Share (% of Initial Value)	Depreciation Coefficient (β)
SMART	72.1%	0.9
Diesel	94.5%	0.6

As evident from the table data, a diesel power plant loses approximately 94.5% of its initial value after 13 years, whereas this figure is about 72.1% for a SMART power plant. This substantial difference of 22.4% clearly demonstrates the significant advantage of SMART technology in preserving asset value over time. This finding confirms that in long-term economic calculations, focusing solely on initial capital costs can be misleading, and depreciation costs must be considered as a decisive factor in technology selection.

The MCMC algorithm was run with 10,000 iterations and a burn-in period of 200 samples. The purpose of this run was to sample the system's key parameters and calibrate the model based on uncertainty data. The acceptance rate of the algorithm in this run was calculated to be 23.4%, which falls within the desirable range (typically between 20% and 40%) for MCMC algorithms and indicates an efficient proposal function.

The posterior distribution of the parameters, representing our updated belief about their values after observing the data, is displayed as a histogram in Fig. 5, (using the wind distribution as an example). Since the MCMC samples are correlated after the burn-in period, the weighted resampling method was used to extract 2000 independent and effective samples.

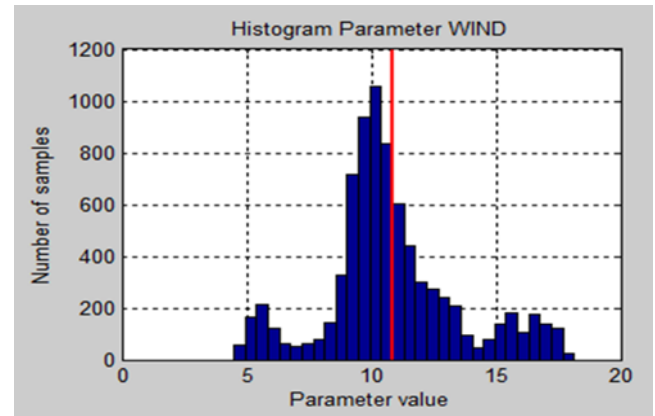


Fig. 5. Histogram plot of the extracted wind output from the MCMC outputs.

The results obtained from the MCMC algorithm indicate that:

1. **Inherent Uncertainty:** The obtained 95% credible intervals confirm the existence of inherent uncertainty in parameter estimation. This uncertainty stems from noise in the primary data and the modeling process itself.

2. **Prudent Estimation:** The mean value for load demand is 656 MW, which is higher than the initial mean of 510 MW. This indicates that, by accounting for uncertainties, the model tends to produce a more conservative and higher estimate of future demand in order to mitigate the risk of power shortage.

3. **Significance of Fuel Prices:** The estimated price for nuclear fuel is \$65.8, which is higher than the initial value of \$64.4, while the diesel fuel price has remained within its initial range. This result indicates that the production cost of nuclear power plants could be a highly influential parameter in optimizing the total system cost.

These effective samples, which represent the joint probability distribution of the system parameters, will serve as the primary input for the next phase of the research: Generation Expansion Planning (GEP) optimization. This approach ensures that the optimal expansion plan is reliable not just for a single deterministic scenario, but for a spectrum of probable parameter values. One of the primary objectives of this research is the minimization of the total energy system cost over the 13-year planning horizon. The total cost comprises two major components: the investment cost in new capacity and the penalty cost resulting from energy shortfall. The optimization results demonstrate that the proposed algorithm has effectively established an optimal balance between these two costs through efficient capital allocation, leading to a significant reduction in the system's cumulative cost.

The system's cumulative cost was calculated using an annual discount rate of 5%. As can be seen in Figs 6-7, the cumulative cost trend is upward but with a decreasing slope, indicating the gradual impact of optimal investments in reducing the system's future costs.

Total Cumulative Cost: 1,289,000k\$ (at a 5% discount rate) by the end of the 13th year (Fig. 6).

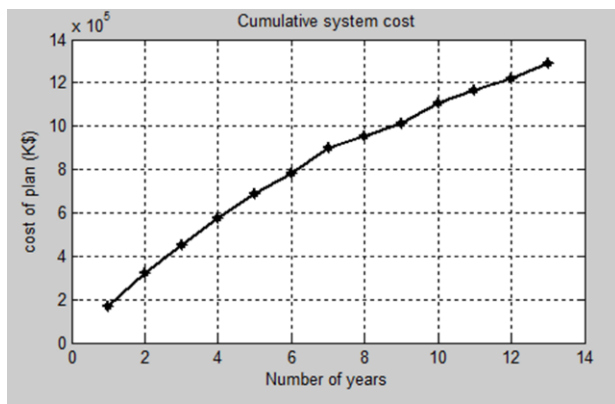


Fig. 6. Cumulative System Cost

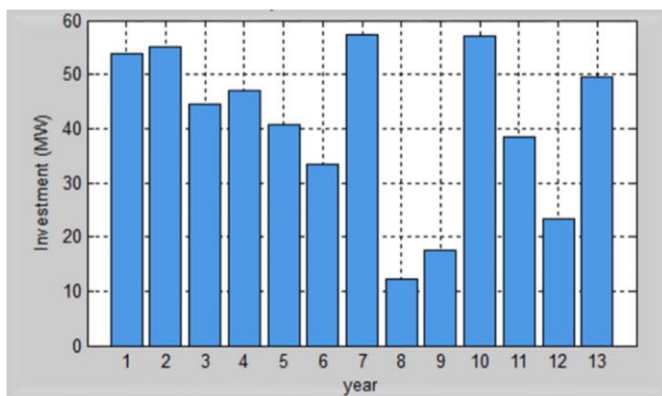


Fig. 7. Optimal Annual Investment

Annual Investment: Between 10 and 60 MW (depending on demand growth and LOLP).

Final System Capacity: 1.32 times the peak demand.

System Efficiency: 78.3% at the end of the 13th year (ratio of supply to actual demand).

System Reliability in the Base Scenario (without BESS):

Based on the results from the capacity expansion optimization, the Loss of Load Probability (LOLP) index was calculated for each year of the 13-year period. The trend of this index is displayed in

Fig. 8.

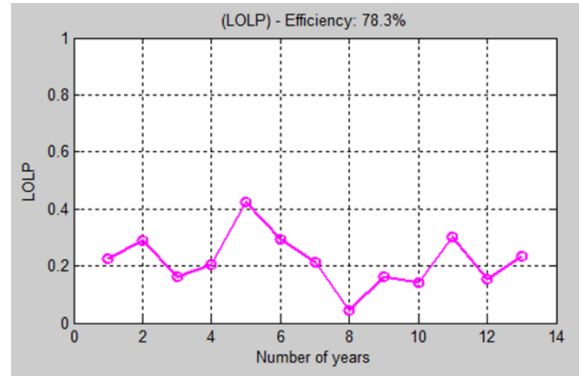


Fig. 8. Yearly LOLP Variations in the Base Scenario

Analysis of Fig. 8 indicates notable fluctuations in the reliability index over the planning horizon. In the early years, the LOLP remains at its lowest level, primarily due to modest demand and ample installed generation capacity. As load demand gradually increases, however, the margin between available capacity and peak demand diminishes, leading to a rising trend in the LOLP.

The average LOLP index over the 13-year period was calculated to be 0.22. Furthermore, the maximum LOLP value occurred in Year 5 and was 0.424, indicating the most critical reliability condition in the system under study.

To better understand the overall long-term impact of BESS on system stability, the cumulative LOLP index over the 13-year period was calculated for both scenarios. This index is essentially the sum of the annual LOLP values and serves as a measure of the "accumulated risk" over the entire period. The results of this calculation are presented in Fig. 9.

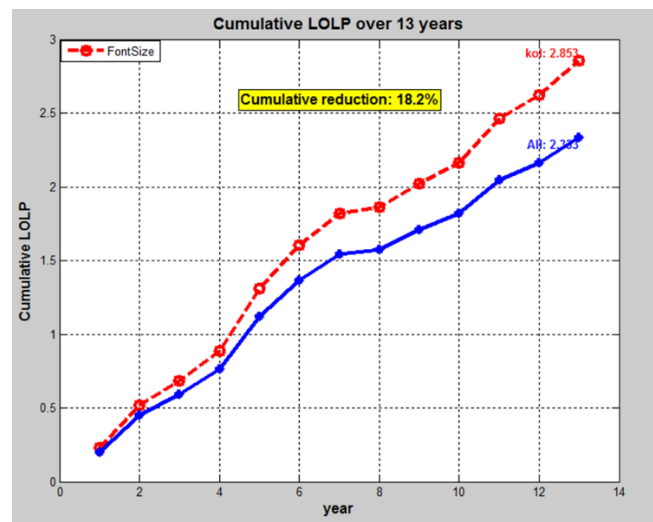


Fig. 9. Comparing Cumulative LOLP Across the Two Scenarios

Comparison of the cumulative LOLP index in scenarios with and without BESS (as shown in Fig. 9), confirms that the cumulative LOLP curve for the BESS scenario (blue) has a gentler slope, indicating a slower risk accumulation rate. By year 13, the total cumulative LOLP is 2.853 in the baseline scenario (red) and 2.333 with BESS, an 18.2% reduction in cumulative system-outage risk due to energy storage.

As mentioned, the proposed hybrid energy system incorporates multiple sources: nuclear power for reliable base load, solar and wind power for renewable energy generation, batteries to manage

fluctuations in real time, and diesel generators can also be used as additional flexible sources to support remote areas. This synergistic configuration, by taking advantage of the geographical flexibility of storage systems that allow them to be located near renewable sources and the rapid response of diesel backups, increases grid stability while promoting clean energy adoption and reducing dependence on fossil fuels.

Payback period analysis:

The discounted cumulative DCF analysis indicates a payback period of more than 12 years. While this represents a long-term capital commitment, it is consistent with the nature of large-scale energy infrastructure projects. Crucially, positive and growing cash flows in the final years predict strong financial performance beyond the payback period, contributing to a strong NPV and ROI.

Based on the financial analysis conducted and taking into account the modeled uncertainties, the project's rate of ROI was calculated at a discount rate of 9%, equal to 15.11%. This value indicates the acceptable economic attractiveness of the project.

The payback period analysis shows that the project will reach the break-even point in a period of more than 12 years. Although this period is relatively long, it is economically justified given the project's useful life of more than 13 years and positive ROI.

5. Conclusions

This study has achieved significant results by providing a comprehensive framework for planning the development of hybrid nuclear-renewable power plants under uncertainty and showing the effects of energy storage batteries. Key findings show that an approximate optimal mix of 73.3% nuclear, 13.2% wind, and 13.5% solar can simultaneously achieve economic and reliability objectives. Simulations performed using a four-step framework including uncertainty modeling, sampling, and optimization, and the use of energy storage batteries in MATLAB software showed that this hybrid mix resulted in a 33% reduction in the LOLP index of the base scenario (to less than 2.85%) and a 18.5% reduction in the second scenario (2.33%) compared to the base scenario. This change was achieved with only 20 MW of storage in the first year at a 10% annual growth rate. Analysis of various parameters showed that the price of diesel fuel had the largest change compared to its initial average and also the largest percentage for the cumulative lost value of the power plant at the end of the study period compared to the first day was related to the diesel power plant with a depreciation coefficient of 0.9, which is a reason to use nuclear power plants instead of this old, expensive and polluting technology. Also, fluctuations in production in renewable sources have the greatest impact on the reliability of the system. Therefore, in this thesis, to eliminate these fluctuations, it was shown that by proposing the use of energy storage batteries along with renewable sources, this will significantly increase the reliability to 97.5%, and, as a result, will lead to the stability of the system. In addition, the use of hybrid nuclear and renewable power plants with a pollution rate close to zero will lead to the achievement of the objective function. The only issue that exists regarding pollution is nuclear waste that needs to be addressed. This waste, which is mainly plutonium, can be kept in the power plant area for a few years to reduce its emissions, then it can be buried under mountains, Neogene sediments, and sandstone. The limitations of this study—including discrete annual modeling (rather than continuous), absence of inter-year dependence, the assumption of linear growth for BESS power, treatment of network parameters as constant over time, and the exclusion of operational fuel costs and nuclear waste management for nuclear power plants—may be addressed in future research. For future research, the development of this model is suggested in four directions: (i)

integration with electricity market mechanisms to enable more realistic simulations; (ii) incorporation of more precise environmental criteria, such as carbon footprint; (iii) application of artificial intelligence methods for real-time optimization, and. (iv) applying some scenario reduction techniques to balance fidelity and tractability, as the number of uncertain variables increases sharply. Furthermore, examining the impact of emerging technologies, including SMART and advancements in energy storage systems, could further strengthen the findings of this study.

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