Wind Generation Investment Planning in a Double Auction Market: A Mathematical Bi-level Approach

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Wind generation investors face multiple problems that could cause different outcomes in their investment problems. In these conditions, the multi-level mathematical models could help them to make a valid decision considering different incompatible objectives in their game. In this paper, a bi-level model is proposed to help a wind power investor to find the optimal plant considering required transmission line reinforcements. This model consists of a double auction market clearing problem in the lower level and the wind generation investment problem in the upper level. The bi-level model is converted to a Mathematical Program with Equilibrium Constraint (MPEC) that tries to solve the investor's problem with consideration of generation uncertainties in a double auction day-ahead pool market. The market model is based on a stepwise supply function offering a structure for maximization of the market Social Welfare (SW) based on given offers and bids. Also, the ratio of social welfare to the market revenue will be investigated for analysis of wind power generation effect on the market parameters. The continuous variables are used for transmission reinforcements in the proposed model instead of common binary ones because of their considerable advantages. The continues variables could show fractional enhancements if needed in any transmission line which could be implemented by FACTS devices. The Garver network and 14-bus IEEE test systems were modeled in GAMS software for presenting the model functionality.

keywords: *Wind generation investment equilibrium, transmission expansion planning (TEP), mathematical program with equilibrium constraints (MPEC), double auction market, bi-level modelling.*

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NOMENCLATURE

f_{nm}	Line capacity from bus n to bus m [MW]
Q^w_n	Invested wind power at bus n [MW]
q^G_{ils}	Accepted generation of unit i at time block l and scenario s [MW]
q^D_{dls}	Accepted consumption of demand d at time block l and scenario s [MW]
θ_{lns}	Bus angle of bus n at time block l and scenario s [rad]
γ_{nm}	Annualized cost of transmission expansion for each MW at line $n-m$ [\$/MW]
$\bar{K}^w$	Available budget for wind power expansion [\$]
$\underline{f}_{nm}$	Existing capacity of line $n-m$ [MW]
$\bar{f}_{nm}$	Accessible capacity expansion of line $n-m$ [MW]
X_{nm}	Reactance of line $n-m$ [Ω]
ϕ_s	Probability weighting factor of each scenario
a^W_{lns}	Wind farm utilization coefficient for different scenarios and time blocks at bus n
$\bar{Q}^W_n$	Maximum accessible capacity expansion of wind power for bus n [MW]
λ^G_{il}	Offered price of unit i at time block l [\$/MWh]
λ^D_{dl}	Bid price of demand d at time block l [\$/MWh]
σ_l	Time block durations [h]
Q^G_{il}	Offered production block at time block l by unit i [MW]
Q^D_{dl}	Demand bid offered by load d at time block l [MW]

1. INTRODUCTION

Wind farms are usually far away from the rest of the network. So, the wind power investment problems could not be solved without consideration of the transmission expansion problem solutions, generally. Accordingly, providing a model that could solve these problems together could be helpful and applicable. Furthermore, it should be considered that wind power expansion is carried out in an environment where the other players in the market (like the Independent System Operator (ISO), other producers, etc.) want to optimize their own objectives. Therefore, the model that could consider the impact of other players could be more realistic and reliable. The most important player of a regulated system is the ISO whose aim is to maximize the SW of the market in different conditions. So, a realistic wind investment problem is generally a bi-level model that considers two agents that have critical roles in wind power expansion. Multi-stage modeling is a common modeling method in hierarchical structure game like power system problems [1, 2] and a variety of different problems solved in recent years. Namely, [3] used it in EV facility allocating problems and [4,5] in mathematical modeling problems and [6] for solving a two-stage strategic bidding problem. The investment is based upon a leader-follower scheme (Stackelberg game) in which the leader wants to optimize its objective subject to its constraints and the followers' decisions, in it. The decisions of the leader determine the feasible area for the follower, and the decisions of the follower have the main role in determining the optimal condition [7]. MPEC (Mathematical Program with Equilibrium Constraint) and EPEC (Equilibrium Problem with Equilibrium Constraint) mathematical modeling methods are popular tools for the mentioned type of games for problems solved using heuristic methods before. References [8, 9] used the mentioned method for modeling electricity market interactions and [10] applied the EPEC method for helping generation expansion planners. In [11], an MPEC model was proposed for modeling the preventive maintenance scheduling problem. Also, the same authors used another Wind farms are usually far away from the rest of the network. So, the wind power investment problems could not be solved without consideration of the transmission expansion problem solutions, generally. Accordingly, providing a model that could solve these problems together could be helpful and applicable. Furthermore, it should be considered that wind power expansion is carried out in an environment where the other players in the market (like the Independent System Operator (ISO), other producers, etc.) want to optimize their own objectives. Therefore, the model that could consider the impact of other players could be more realistic and reliable. 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As a result, a bi-level model that could solve a wind power planner problem under a double auction market is needed. The model uses continuous variables for transmission expansion planning based on their advantages. In the end, the bi-level model is transformed into a MILP model using the SOS1 linearization technique for its complementarity constraints. The paper illustrates the model descriptions for each level in the model assumptions section. Additionally, the transformation of the bi-level model into a one-level MILP level is discussed there. Then, the model is applied to the 6-bus Garver network and the IEEE 14-bus case study showing the model functionality. The conclusions of the work are presented at the end of the paper.

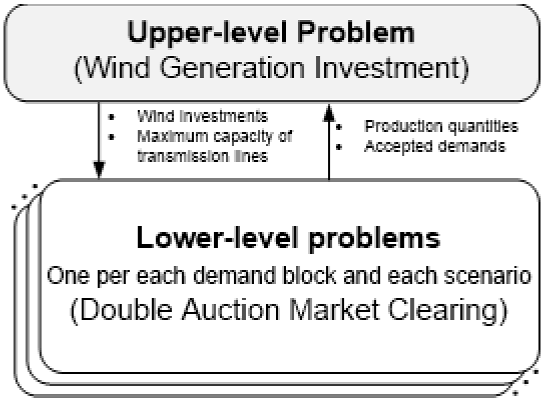

Fig. 1. The bi-level structure of the wind power investment problem.

2. MODEL ASSUMPTIONS

The proposed model presents a wind power investor that maximizes their revenues and minimizes the turbine's installation cost and the required transmission line investments. Moreover, the investor takes into account the ISO decisions for market clearing in the lower level. This helps a wind power investor to obtain reliable responses in a market environment; therefore, the proposed bi-level model considers the wind power investor's problem at the upper level and the market-clearing problem of the ISO in the lower level. Figure 1 shows the interaction between these major players using a schematic model.

A. Upper level (Wind power investor problem)

This stage considers the wind investment objective for finding the best size and the optimal location of the wind farm. In some countries, considering the investment cost of grid connection for wind farms, is in responsibility of the renewable generation investor. Therefore, it should be considered as a part of total investment costs in this type of utilities. Consequently, the objective function of this layer consists of the minimization of wind power investment costs, the transmission expansion or reinforcement costs minus the total revenue of the wind generation in different scenarios and time blocks as presented in (1). For calculating the transmission expansion cost, the quantity of reinforced capacity (f_{nm}) has been subtracted from the existing capacity ($\underline{f}_{nm}$) of the power lines and has multiplied into the annualized cost of each transmission line per each MW. The formulation of this level is given below:

$$\begin{aligned}&\min_{f,Q^W}\left\{\sum_{n,m\in\Omega_n} y_{nm}(f_{nm}-f_{-nm})\right\}+\sum_n y^W_n Q^W_n\\&+\sum_{l,s}\varphi_s\sigma_l\left[\sum_{i\in G}\lambda^G_{il}q^G_{ils}-\sum_{d\in D}\lambda^D_{dl}q^D_{dls}\right]\end{aligned}\tag{1}$$

$$\underline{f}_{nm}\leq f_{nm}\leq\bar{f}_{nm}\quad\forall n,m\in\Omega_n\tag{2}$$

$$0\leq Q^W_n\leq\bar{Q}^W_n\quad\forall n\tag{3}$$

$$\sum_n\gamma^W_n Q^W_n\leq\bar{K}^W\tag{4}$$

Equation (2) limits the maximum capacity of transmission lines between the existing capacity ($\underline{f}_{nm}$) and the accessible capacity expansion ($\bar{f}_{nm}$). If the expansion could not be possible, the upper and lower limits are the same. The next equation, (2), determines the available capacity of wind power installation at the different buses of the network. It emphasizes that the reachable capacity of wind farms is distinctive at different locations. Finally, constraint (3) bounds the available investment budget for the wind investor.

B. Lower level (Dual Auction Market Clearing)

This level, as a constraint for the upper-level, wants to maximize the true social welfare of the dual auction market for each scenario at a time block as an objective of the market operator. It should be mentioned that wind power generators offer their energy at zero price, which is consistent with most real-world electricity markets [24,25]. So, their production has not been considered in calculating of the lower-level objective function.

$$\min_{q^G,q^D,\theta}\{\sum_{i\in G}\lambda^G_{il}q^G_{ils}-\sum_{d\in D}\lambda^D_{dl}q^D_{dls}\tag{5}$$

$$subject\ \ to:$$

$$\begin{aligned}&\sum_{d\in D_n}q^D_{dls}-\sum_{i\in G_n}q^G_{ils}-a^W_{lns}Q^W_n\\&+\sum_{m\in\Omega_n}\frac{1}{X_{nm}}(\theta_{lns}-\theta_{lms})=0\ \ \forall n:p_{lns}\end{aligned}\tag{6}$$

$$0\leq q^G_{ils}\leq Q^G_{il}\ \ \forall i\tag{7}$$

$$0\leq q^D_{dls}\leq Q^D_{dl}\forall d\tag{8}$$

$$\frac{1}{X_{nm}}(\theta_{lns}-\theta_{lms})\leq f_{nm}\ \ \forall n,m\in\Omega_n\ \ :\ \ v_{lnms}\tag{9}$$

$$-\pi\leq\theta_{lns}\leq\pi\ \ \forall n:\underline{\xi}_{lns},\bar{\xi}_{lns}\tag{10}$$

$$\theta_{lns}=0\ \ \forall n=1:\xi^1_{ls}\}\ \ \forall l\tag{11}$$

Constraint (5) forces the load and generation balance at each bus. Equation (7) shows that the accepted generation of each unit could not be more than its offered energy block for each time block. The next equation, (8), limits the accepted energy bid from demands, in a similar way. Equation (9) considers the maximum available flow for each transmission line. It should be mentioned that f_{nm} is a variable for the upper level, but it is a constant for the lower level. The next constraints, (10) – (11), enforce the voltage angle limitations for each bus

C. MPEC

For making the bi-level optimization problem solvable using commercially available branch-and-cut solvers, the given equations for the upper and lower levels should be converted into a single-stage MPEC program using two common methods [26]. Both methods provide equal optimization models and solutions and could be used instead of each other. One of these methods is the primal-dual method, and the other one is the KKT derivations of the lower level. Generally, the mentioned methods produce non-linear equations that could not be proper for mathematical solutions. One of the major non-linear forms in these models are the complementarity constraints of the KKT derivation that could be shown in the form 0<ab>0. These equations did not exist in the first method (primal-dual conditions), and they are equal to strong duality conditions in the primal-dual technique. Using these methods, the bi-level problem could be transformed into the one-stage problem that is known as MPEC and it could be solvable using mathematical solvers. The approach that is used in this paper for producing MPEC is the KKT method. As a result, the mentioned bi-level wind power investment problem is converted into a one-stage optimality problem in set (c) that contains upper and lower constraints and the KKT conditions. The new complementarity constraints that belong to the non-equality constraints in the lower level are merged with the upper-level constraints. Equation (12) comprises the objective function of the upper level as the objective function of the single-level problem. The other constraints of both levels of the bi-level model are the other parts of the new problem. Additionally, equality constraints (16-17), are the dual constraints of the lower-level problem. The complementarity conditions for non-equality constraints are shown in (21) – (27).

$$\begin{aligned}&\min_{f,Q^W}\{\sum_{n,m\in\Omega_n}\gamma_{nm}(f_{nm}-\underline{f}_{nm})+\sum_n\gamma^W_n Q^W_n\\&+\sum_{l,s}\varphi_s\sigma_l[\sum_{i\in G}\lambda^G_{il}q^G_{ils}-\sum_{d\in D}\lambda^D_{dl}q^D_{dls}]\end{aligned}\tag{12}$$

$$\begin{aligned}&subject\ \ to:\\&\underline{f}_{nm}\leq f_{nm}\leq\bar{f}_{nm}\forall n,m\in\Omega_n\end{aligned}\tag{13}$$

$$0\leq Q^W_n\leq\bar{Q}^W_n\ \ \forall n\tag{14}$$

$$\sum_n\gamma^W_n Q^W_n\leq\bar{K}^W\tag{15}$$

$$\begin{aligned}&\sum_{d\in D_n}q^D_{dls}-\sum_{i\in G_n}q^G_{ils}-a^W_{\ln s}Q^W_n\\&+\sum_{m\in\Omega_n}\frac{1}{X_{nm}}(\theta_{\ln s}-\theta_{lms})=0\ \ \forall n:p_{\ln s}\end{aligned}\tag{16}$$

$$\theta_{\ln s}=0\forall n=1,\ \ \forall l,s\tag{17}$$

$$\lambda^G_{il}-p_{l(n:i)s}-\underline{\mu}^G_{ils}+\bar{\mu}^G_{ils}=0\ \ \forall i,l,s\tag{18}$$

$$-\lambda^D_{dl}+p_{l(n:i)s}-\underline{\mu}^D_{dls}+\bar{\mu}^D_{dls}=0\ \ \forall d,l,s\tag{19}$$

$$\begin{aligned}&\sum_{m\in\Omega_n}\tfrac{1}{X_{nm}}(p_{lns}-p_{lms})+\sum_{m\in\Omega_n}\tfrac{1}{X_{nm}}(v_{lnms}-v_{lmns})\\&-\underline{\xi}_{lns}+\bar{\xi}_{lns}+(\xi^{1}_{ls})_{n=1}=0\ \ \forall l,n,s\end{aligned}\tag{20}$$

$$0\le q^{G}_{ils}\perp\underline{\mu}^{G}_{ils}\ge 0\tag{21}$$

$$0\le Q^{G}_{ils}-q^{G}_{ils}\perp\underline{\mu}^{G}_{ils}\ge 0\tag{22}$$

$$0\le q^{D}_{dls}\perp\underline{\mu}^{D}_{dls}\ge 0\tag{23}$$

$$0\le Q^{D}_{dl}-q^{D}_{dls}\perp\bar{\mu}^{D}_{ils}\ge 0\tag{24}$$

$$0\le f_{nm}-\frac{1}{X_{nm}}(\theta_{lns}-\theta_{lms})\perp v_{lnms}\ge 0\tag{25}$$

$$0\le(\pi+\theta_{lns})\perp\underline{\xi}_{lns}\ge 0\tag{26}$$

$$0\le(\pi-\theta_{lns})\perp\underline{\xi}_{lns}\ge 0\tag{27}$$

D. Linearization

As shown before, because of the complementarity constraints, the produced single-stage model is non-linear and creates a non-convex problem that may yield a local optimum. So, the given model should be linearized using proper linearization methods. The common method for linearization of these types of non-linear constraints is replacing them with equivalent disjunctive formulations that contain binary variables and big-M constants. Using integer and binary variables in mathematical optimization models make them complicated for existing solvers and causes some troubles like requirements for frequent regulation of big-M values. Therefore, a new linearization method is applied throughout this paper based on using SOS1 variables and a penalty factor with positive variables. This method has effective benefits and calculation advantages in comparison with other methods for the linearization of complementarity constraints [23]. For instance, the linearization of a complementarity constraint like $0 < a \perp b > 0$ with the SOS1 method, requires the following procedure:

$$\begin{aligned}&a\ge 0,\quad b\ge 0\\&v^{+}+v^{-}=\tfrac{a+b}{2}\\&v^{+}-v^{-}=\tfrac{a-b}{2}\\&v^{+},v^{-}=\tfrac{a-b}{2}\\&v^{+},v^{-}\in SOS1\ \ variables\end{aligned}\tag{28}$$

The SOS1 variable is a type of variable that could have one positive element inside the vectors, and the other arrays should be equal to zero. This type of variable is known for usual mathematical solvers like CPLEX and could be used in the linearization of complex models [27, 28]. However, for dealing with this type of variable, a combined method using a penalty factor is proposed. This approach helps to replace the SOS1 variables with positive ones by considering a penalty function together with the objective function. This method helps the model to be simpler than previous techniques. For this purpose, the mentioned model should be changed as follows for having a proper linearization:

$$\begin{aligned}&\min_{f,Q^{W}}\{\sum_{n,m\in\Omega_n}\gamma_{nm}(f_{nm}-\underline{f}_{nm})+\sum_{n}\gamma^{W}_{n}Q^{W}_{n}\\&+\sum_{l,s}\varphi_s\sigma_l[\sum_{i\in G}\lambda^{G}_{il}q^{G}_{ils}-\sum_{d\in D}\lambda^{D}_{dl}q^{D}_{dls}]\\&L\sum_{k}(v^{+}_{k}+v^{-}_{k})\}\end{aligned}\tag{29}$$

$$\begin{aligned}&subjectto:\\&v^{+}_{k}+v^{-}_{k}=\tfrac{a_k+b_k}{2}\end{aligned}\tag{30}$$

$$\begin{aligned}&v^{+}_{k}-v^{-}_{k}=\tfrac{a_k-b_k}{2}\\&a_k\ge 0,\ \ b_k\ge 0,\ \ v^{+}_{k}\ge 0,\ \ v^{-}_{k}\ge 0\end{aligned}\tag{31}$$

If there is a constraint that was not satisfied with the approximate method, the variables of the mentioned constraint could be returned to the SOS1 type. Using this approach can considerably decrease the model complexity.

3. CASE STUDIES

As declared before, the proposed model could solve the wind power generation expansion planning problem with its required transmission expansions considering a double auction market cleared by the ISO. The mentioned capabilities of the model are examined using two different case studies implemented on two standard networks. The first case, $Case_1$, finds the expansion plan of wind power generation and transmission lines of the Garver test system. In the next case study, the proposed model is implemented on the IEEE 14-bus test system as a larger network.

A. Garver Network

One of the most popular test systems in TEP studies is Garver 6-bus test network [29]. The 6-bus test grid initially has six transmission lines and five loads. Three existent generation units could supply about 660 MW, and the existing lines could be enhanced or built between candidate buses. The modified system with proposed locations for wind farms is shown in Figure 2. As can be seen from the figure, the Garver system has an isolated bus that could be considered as a new wind farm location. For having a comparison, two candidate wind farms are proposed for this case. One is considered on the isolated bus, and the other one on bus 4, which has connections with other buses of the system. The capacity of these wind farms is about 600 MW, at most. The considered weighting factors corresponding to each time block are 1200, 3600, 2400, and 1560, derived from the load duration curve of the planning year. Besides, we introduce three scenarios for wind power with the probability weighting factors and derating factors of each scenario presented in Table 1 [20, 30]. Furthermore, the price and energy bids for loads and offers for generation units in a five-time block have been presented in tables 2 and 3, respectively. Table 2 provides the blocks of the energy and the corresponding price bid for the conventional production units. It should be noted that the generating units use their total capacities provided in Table 3 for offering to the market at each time block. These quantities are assumed fixed throughout the planning horizon.

The model proposes two locations for the installation of wind farms. The size of the wind power plant proposed to be at bus 4 is about 215.93 MW, and, for bus 6 is about 418.16 MW, with a total wind power investment cost of 63.41 M$. Furthermore, the results show that the transmission line's enforcement should be done in four lines with a cost of 15.42 M$. The results propose 196.47 MW enforcement for lines 2-6, 144.21 MW for 4-6, 45.96 MW for 3-4, and 4.83 MW for 3-5, at the end. The existing capacity of line 3-4 is about 82 MW, and the required enforcement could be done using other routine methods for enforcement of lines that has less cost that installation a new line in the mentioned corridor. For example, it could be applied using FACTS or using a bundled line. The required support for lines 3-5 is

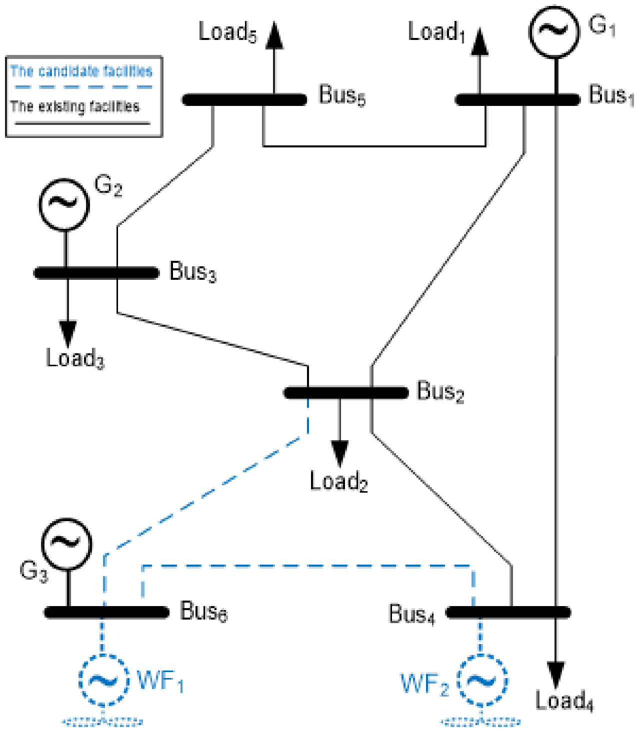

Fig. 2. The Garver Test System

very low, so it should be done with the mentioned methods, too. Figure 3 shows the social welfare percentage rather than the incoming revenue from buying demand. It shows that this percentage increases in the latest time blocks into 100% at the final time block. The reason is that the total generation in the mentioned time block was supported by the wind type plants because of their low-price offers, as mentioned before in section 2.2.

B. IEEE 14-Bus Test System

This section considers the IEEE 14-bus test system including eleven loads, five generating units (G1 to G5), and twenty lines as existing facilities. Five demand blocks with durations of 1000, 3000, 3000, 1000, and 760 hours for each time block are considered. The load level in the first demand block is the same as that in the original model [31] raised by 50%. The considered

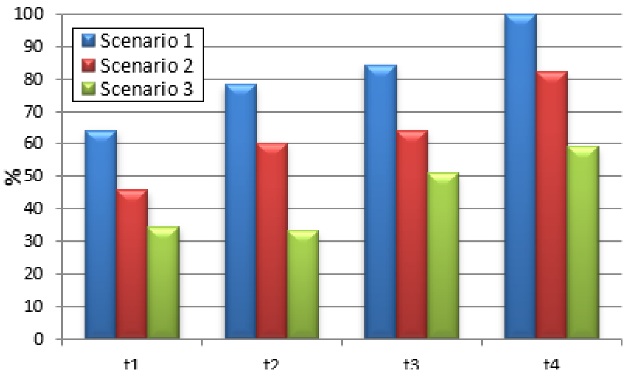

Fig. 3. The ratio of social welfare rather to market revenue

Table 1. Derating factors for wind farms within different scenarios

		Scenario1 (15%)	Scenario2 (70%)	Scenario3 (15%)
Time block1	Bus4	0.436	0.267	0.122
	Bus6	0.411	0.232	0.099
Time block2	Bus4	0.401	0.223	0.122
	Bus6	0.456	0.267	0.095
Time block3	Bus4	0.365	0.223	0.112
	Bus6	0.382	0.215	0.093
Time block4	Bus4	0.351	0.194	0.095
	Bus6	0.364	0.204	0.105

Table 2. Demand blocks [MWh] and price bids [$/MWh] of the loads

	Time block1		Time block2		Time block3		Time block4	
	Qty	Price	Qty	Price	Qty	Price	Qty	Price
Load1	60	30	48	28	36	26	24	24
Load2	180	34	144	32	108	30	72	28
Load3	30	25	24	24	18	23	12	22
Load4	120	30	96	27	72	24	48	21
Load5	180	34	144	30	108	26	72	24

Table 3. Offer prices for regular generating units at different time blocks [$/MWh]

	Capacity [MW]	Time block1	Time block2	Time block3	Time block4
G1	150	20	18	16	14
G2	360	24	23	22	21
G3	150	20	16	14	10

load levels and their related bid prices in different demand blocks are given in Table 4. Additionally, Table 5 includes the offer prices and size data for existing generation units. The proposed wind farms are considered to be located at either bus N1 or bus N6 as it could be seen in Figure 4. According to the uncertain behavior of the wind generating units, six scenarios with identical weightings (i.e., 1/6) have been considered in the case. The wind generation derating factor for the wind farm on N1 has given in Table 6 for each scenario [32]. The derating factors for the wind farm of N6 is 10 percent more than derating factors of N1 for different scenarios and time blocks. Also, Table 7 contains the data for four new candidate transmission lines.

The model solution proposes two locations for wind power investment in the network, a 161.13 MW wind farm at bus N1 and another one with a capacity of 483.7 MW at bus N6, with a

Table 4. Demand blocks [MWh] and price bids [$/MWh] of different loads

	Load1		Load2		Load3		Load4		Load5		Load6	
	Qty.	Price	Qty.	Price	Qty.	Price	Qty.	Price	Qty.	Price	Qty.	Price
Time block 1	49	35	214	35	108	35	17	35	25	40	66	38
Time block 2	41.3	31	178.1	33.2	81	33.5	14.1	31.5	18.7	36	47.6	34.2
Time block 3	31	28.5	105.1	31.5	57	31.4	8.3	29	12.4	33.4	31.7	30.6
Time block 4	28	26.1	91	28	46	26	7.7	27.4	10.7	31.5	27.2	39.2
Time block 5	25.4	24.5	83.4	24.5	43	24.5	7	24.5	9	28	24.1	26.1

	Load7		Load8		Load9		Load10		Load11	
	Qty.	Price	Qty.	Price	Qty.	Price	Qty.	Price	Qty.	Price
Time block 1	21	40	8	38	14	40	31	38	34	38
Time block 2	16	36	5.7	36.2	10.1	36	22	36.2	25.5	36.2
Time block 3	13.8	30	4	34.6	8.5	30	11.5	30.6	18.4	32.6
Time block 4	11	29	3.2	30.7	7.1	29	8.3	28	16.5	30.1
Time block 5	9.5	28	3	26.1	6.3	28	7.8	26.6	15.8	26.3

Table 5. Offer prices for regular generating units at different time blocks [$/MWh]

	Capacity [MW]	Time block1	Time block2	Time block3	Time block4	Time block5
G1	200	25	23	21	19	16
G2	150	24	23	22	20	18
G3	130	20	18	16	14	12
G4	150	35	32	30	29	29
G5	100	32	28	27	25	23

Table 6. Factor within different scenarios and time blocks for wind farm on N1

	Time block 1	Time block 2	Time block 3	Time block 4	Time block 5
Scenario 1	0.469	0.473	0.421	0.514	0.507
Scenario 2	0.354	0.335	0.297	0.402	0.400
Scenario 3	0.264	0.259	0.234	0.301	0.321
Scenario 4	0.199	0.192	0.179	0.223	0.225
Scenario 5	0.130	0.125	0.120	0.144	0.116
Scenario 6	0.067	0.060	0.067	0.073	0.042

Table 7. Data for the candidate transmission lines

Candidate line	N1-N3	N1-N9	N2-N10	N6-N14
Capacity [MW]	168	168	168	168
Annualized cost [M$]	2.2	3.0	2.0	2.5
Susceptance [S]	500	768	521	909

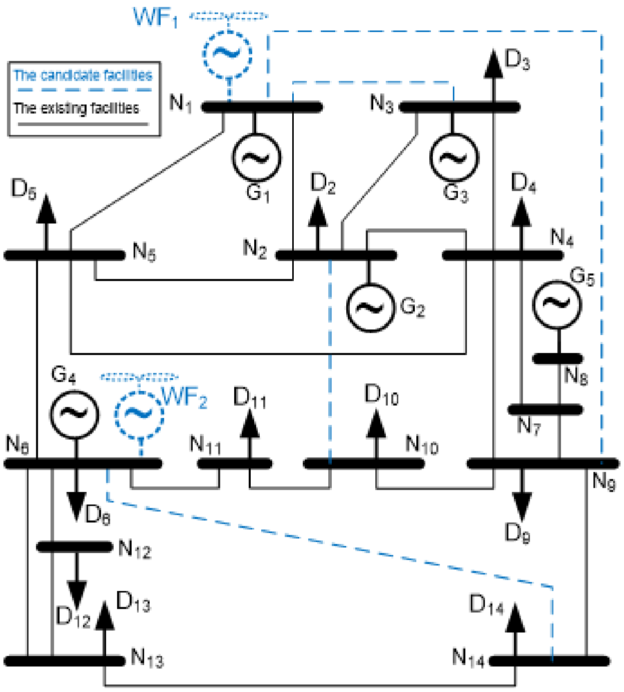

Fig. 4. The network of the IEEE 14-Bus test system

Table 8. The required reinforcements for different transmission lines

Line	Current Capacity	Required reinforcement
N1-N3	-	91.94
N1-N9	-	71.05
N2-N10	-	43.41
N6-N14	-	67.14

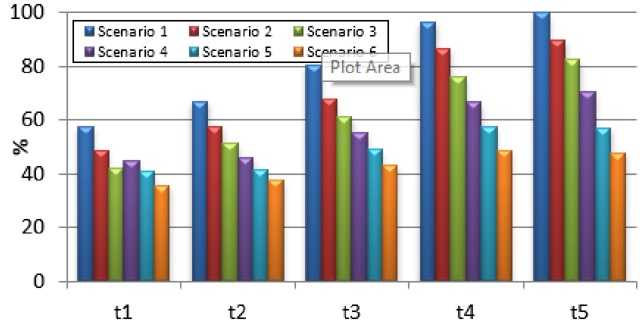

Fig. 5. The ratio of social welfare rather to market revenue

total wind power investment cost of 64.48 M$. Furthermore, it recommends four transmission expansions considering the new condition of the network, as mentioned in Table 8, which results in about 0.25 M$ of annualized cost. In addition, the graph chart that shows the social welfare related to market revenue has been presented in Figure 5. The MILP model of MPEC is solved using CPLEX 12.6 [33] under General Algebraic Modeling System (GAMS) [34] on an Intel Core i7-2620M computer with 4 processors at 2.7 GHz and 4 GB of RAM. The relative optimality gap was enforced to be zero for all the cases.

4. CONCLUSION

The modification of a bi-level compact mathematical model for wind power investment problems has been done in this paper. Without using any decomposition or iteration techniques, this model considers both sides of a wind power investment problem containing transmission and wind farm planning programs in a double auction market operated system. Illustrative examples, including the Garver network and the IEEE 14-bus test systems, show the accurate results of the proposed methodology. Also, the results explain that using continuous variables could help the investor to catch the best solutions for supporting transmission lines. Solutions found from tools such as the one presented throughout this paper should not be applied blindly in actual planning exercises, but rather help planners gain more insight into the agent's expected behavior.

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