

Coordinated Generation and Transmission Expansion Planning with Optimal Wind and Thermal Power Integration

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In this paper, coordinated transmission and generation expansion planning is presented and a comprehensive approach is proposed for determining optimal wind energy integration. In this way, a multi-state model is introduced for wind farms and correlation between wind farms is considered with a copula method. Optimal wind power integration is determined with regard to desired reliability level. In addition, thermal power integration is obtained for improvement of reliability in the presence of wind farms. Thus, optimal combination of new wind and thermal power is obtained considering technical and economic factors. The impacts of wind speed, correlation between wind farms, reliability level and emission penalty are evaluated on wind power integration and a Benders decomposition method is utilized that can be easily implemented on real large cases. The proposed method is applied to IEEE 24-bus and 118-bus test systems and its performance is demonstrated by evaluating the more influencing factors on wind farms integration. © 2018 Journal of Energy Management and Technology

keywords: Copula, Multi-state model, Wind farms, Expansion planning, Correlation.

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NOMENCLATURE

Indices:

i Index of buses

ij Index of lines

ui Index of units in bus i

wi Index of wind farms in bus i

s Index 's' is used for showing variables in scenario 's'

Sets:

Γ Set of all Lines

B Set of all buses

UI Set of units in bus i

WI Set of wind farms in bus i

S Set of scenarios

Parameters and variables:

Cl_{ij} Investment cost of line ij (\$)

C_{ui} Investment cost of thermal unit u in bus i (\$)

C_{ui}^0 Operational cost of thermal unit u in bus i (\$)

C_{ui}^e Emission cost of unit i (\$)

T_s Duration of scenario s (s)

A^T Lines and units connection matrix

P_{ui}^0 Generation of existing thermal unit u in bus i (MW)

P_{ui} Generation of new thermal unit u in bus i (MW)

P_{wi} Generation of wind farm w in bus i (MW)

$P_{ui,min}$ Minimum generation of thermal unit u in bus i (MW)

\bar{P}_{ui} Nominal power of thermal unit u in bus i (MW)

\bar{P}_{wi} Nominal power of wind farm w in bus i (MW)

f_{ij}^0 Active power flow of existing line ij (MW)

- f_{ij} Active power flow of new line ij (MW)
- \bar{f}_{ij} Maximum active power flow of line ij (MW)
- $\bar{\delta}_{ij}$ Maximum voltage angle difference between buses i and j (rad)
- Ls_i Load shedding in bus i (MW)
- Q_{uij} Contingency state for line ij
- Q_{ui} Contingency state for thermal unit u in bus i
- m_{ui} Binary decision variable of new thermal unit u in bus i
- m_{wi} Binary decision variable of new wind farm w in bus i
- a_i, b_i Coefficients for operational cost of thermal unit i (\$/MWh)
- c penalty cost of emissions (\$/MWh)
- q Interest rate item [a] Operational life of units
- $E\bar{E}NS$ Upper limit for EENS

1. INTRODUCTION

Because of the importance of environmental pollution in recent years, wind energy is widely integrated in power systems. According to the large investment cost of wind farms, a comprehensive study should be dedicated for determining the optimal capacity and location of them. In this way, reliability evaluation and uncertainty of wind speed are always taken into account due to variations of wind speed. Different aspects of wind farms integration are considered in the researches. Wind farms are modeled with different methods in the researches. Fitting a real wind speed density function to Weibull distribution is presented in [1] and the shape and scale parameters of Weibull distribution are calculated. In [2], a bivariate model is obtained by wind speed and wind directions historical data; however, if numerous wind farms are considered, it is difficult to utilize this model. The markov model [3] and the multi-state model [4,5] are also considered for wind farms. In [6], a multi-state model is introduced and capacity outage probability and frequency tables are prepared for reliability evaluation in the presence of wind farms. In [7], multi-state model is considered for load and wind speed. Auto Regressive Moving Average (ARMA) method is utilized for the expansion of wind speed data in [8]. Correlations between wind farms and between load and wind speed are effective factors in wind farms penetration that are taken into account in the researches. In [9], a time-shift method and in [10], Cholesky decomposition method is considered. An efficient method for considering correlation is copula that is utilized in [11–13]. Optimal copula is determined in [11]. A pair copula method is proposed in [12]. In [13], a copula method is utilized for correlation between load and wind speed. In these researches, system reliability is evaluated in the presence of wind farms but the expansion planning is not taken into account.

The fluctuations of wind speeds is are considered in the papers. In [14], different uncertainties of wind farms are investigated with a sensitivity analysis. The methods for wind power intermittency mitigation are reviewed in [15]. More influencing factors on wind power curtailment are discussed in [16]. In [14–16], wind speed variations are considered but expansion planning is not taken into account.

Wind farms integration is considered in Transmission Expansion Planning (TEP) or Generation Expansion Planning (GEP). In [17–19], wind farms are integrated in simultaneous TEP and GEP. In these researches, wind farms correlations are not taken into account. In [17], some regulations are considered for wind power and stochastic programming is not taken into account. In [18], an iterative process is proposed for coordinated expansion planning of wind farms and transmission lines. The expansion of conventional units is not considered in this reference. In [19], a Mixed Integer Programming (MIP) method is used. Although a scenario reduction technique is utilized, it is difficult to solve the proposed MIP method with large number of equations. In [20–27], wind farms penetration is considered in TEP with uncertainties. In these references, the impact of wind farms is considered in TEP with different methods, however, wind power planning is not considered. In [20], a clustering scenario generation is considered in TEP. In [21], an Imperialist Competitive Algorithm (ICA) is utilized. Monte Carlo is used for creating the wind speed scenarios. In [22], Benders Decomposition (BD) and Monte Carlo methods are utilized for security and reliability evaluation. In [23], wind speed variations are considered by virtual power plants with high failure cost. In [24], a multi-state model is utilized for wind speed and load. In [25], curtailed wind energy is taken into account. In [26], some scenarios are considered for renewable generations. In [27], scenarios are created for wind speeds uncertainty. In some papers, the concentration is on wind speed uncertainty in TEP. In [28], a sequential method and in [29–31], point estimation methods is considered. In [32], uncertainty is taken into account with a BD method and in [33], a defined range is considered for variations of uncertain variables. In [34], uncertainty and robustness are considered simultaneously, however, many approximations and linearization are considered and the accuracy of the solution is reduced.

In some researches, wind farms are integrated in GEP [35–37] and reliability in the presence of wind farms is considered. In these researches, wind farms correlations are not taken into account. In addition, transmission lines are not considered in reliability evaluation (HLII). In [35], a long term generation scheduling is conducted in the presence of wind farms. In this method, types and capacities of generation units are obtained with an optimal power flow problem considering scenarios. In [36,37], short-term constraints are considered in GEP. In [36], scenarios are considered for wind farms and only one equipment failure is taken into account. In [37], penalties are defined for wind power intermittency in a multi-objective problem. In [38], emission penalties are considered to increase wind power penetration.

A presented method should be able to be easily implemented to large-scale systems. In [39], a multi-stage robust optimization approach is proposed for expansion planning with uncertainties. However, it is difficult to solve this method for large-scale cases. A heuristic method is introduced in [40] for considering uncertainty of wind speed in TEP; however, the presented method is a time consuming method due to daily optimal power flow process. In [41], coordinated TEP and GEP is presented with a comprehensive heuristic method however, it is difficult to solve the presented large number of equations.

A. Contributions

Consider a candidate place with high wind speed. What factors do limit the wind power in this place? Does reliability level limit the wind? Do economic reasons limit the wind? These questions are answered in this paper. According to large investment cost

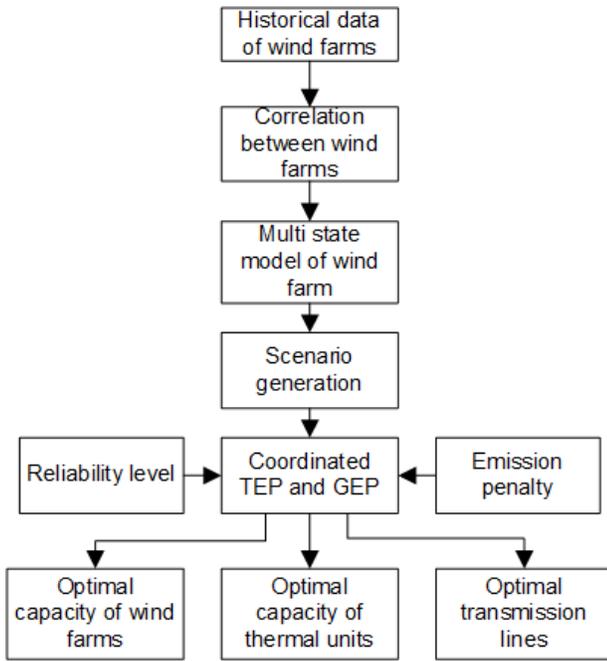


Fig. 1. Schematic of the proposed method

wind farms, determining optimal wind farms integration is very important. In the previous researches, optimal combination of wind farms and thermal power is rarely considered and the impact of correlation and transmission system is not taken into account.

In this paper, correlation between wind farms is considered with a copula method and a multi-state model is introduced for all wind farms. Coordinated TEP and GEP is presented considering wind farms. Therefore, optimal capacity and location of wind power are determined. A limit for reliability level is taken into account and the impact of emission is considered to increase the quota of wind power. The schematic of contributions is presented in Fig. 1. It is shown that the decision variables are location and capacity of thermal units, transmission lines and optimal capacity of wind farms considering desired reliability level.

Scenarios are utilized duo to wind speed uncertainties. A BD method is proposed for composite TEP and GEP that can be easily implemented on the large-scale practical systems. The features of the proposed method and the previous researches are compared in Table 1. It can be seen that more influencing factors on wind farms integration are included in this paper that are not integrated in the previous researches. The main contributions of this paper are itemized in the following:

- A multi-state model is considered for wind farms in coordinated TEP and GEP.
- A copula method is used for correlation between wind farms in the expansion planning.
- The most influencing factors for increasing wind farms integration are presented.
- A BD method is utilized that can be easily implemented on large-scale practical systems.

The remainder of this paper is organized as follows: section 2 considers wind farms correlation. Section 3 discusses a multi-state model of wind farms. Coordinated TEP and GEP is given in section 4. In section 5, the proposed approach is applied to the case studies, and the conclusion remarks are provided in section 6.

2. CORRELATION BETWEEN WIND FARMS

The correlation between wind farms is an important factor in wind power integration. Copula is an efficient method for modelling wind farms dependency [11–13]. A method is introduced for generating correlated random variables in the following subsections.

A. Generating random variables

Random variables of wind speeds are obtained according to Weibull Probability Distribution Function (PDF) and the shape and scale parameters are determined according to historical data. Marginal distribution function of random variables can be obtained by parametric or non-parametric methods [11]. In this paper, Weibull PDF is utilized for wind speed that is shown in (1) and commutative distribution function is obtained in (2).

$$f(x) = \frac{k}{\lambda} \left(\frac{x}{\lambda}\right)^{k-1} e^{-\left(\frac{x}{\lambda}\right)^k} \quad (1)$$

$$F(x) = \int_0^x f(x) \cdot dx = 1 - e^{-\left(\frac{x}{\lambda}\right)^k} \quad (2)$$

where, k and λ are shape and scale parameters, respectively and Γ is Gama function. $f(x)$ and $F(x)$ are probability and commutative distribution functions, respectively.

B. Measuring dependency

Dependency of random variables can be measured by different methods [9–11]. The most common method is the Pearson product-moment correlation coefficient. Rank correlation coefficients, such as Spearman's rank and Kendall's rank correlation coefficients are other typical measures in this field. In this paper, Kendall's tau that is given in (3) is utilized duo to its performance in measuring wind speeds dependency [10].

$$\tau = \frac{2}{n(n-1)} \sum_i \sum_{j>i} \text{sgn} \left\{ (x_i - x_j) (y_i - y_j) \right\} \quad (3)$$

where, n is the number of random variables, x_i and y_i are two sets of random variables. sgn is sign function.

C. Copula method for generating joint distribution function

As explained before, some methods have been utilized for generating dependent random variables [11, 12] and copula is an efficient method in this field [13]. There are different types of copulas, such as Archimedean copulas, Gaussian copulas and extreme-value copulas. In this paper, Archimedean copula is utilized duo to its simplicity and advantages in generating correlated samples of wind speed [18]. Archimedean copula is shown in (4). Gumble generator function is used that is shown in (5). Variable θ , is dependence parameter that is obtained by the estimation methods [11]. For example consider $F(z)$ and $G(u)$ as Weibull distribution functions of two wind sites as shown in (6) and (7). Joint distribution function of these wind regimes is given in (8).

$$C(u_1, \dots, u_n) = \varphi^{-1}(\varphi_1 + \dots + \varphi_n) \quad (4)$$

Table 1. The optimum results for four CSTs

Ref no.	TEP	GEP	Wind farms integration	Wind farms modelling	Formulation	Uncertainty	Reliability or contingency	Correlation	Emissions
[1]				*					
[2]				*					
[3–6]			*	*			*		
[9–13]			*	*			*	*	*
[14–16]			*			*			
[17]	*	*	*		NLP				
[18]	*	*	*		Heuristic				
[19]	*	*	*		MIP				
[20]	*		*		MIP				
[21]	*		*		ICA	*			
[21]	*		*		BD	*	*		
[23]	*		*		MILP		*		
[24]	*		*	*	GA	*	*		
[25]	*		*		MO	*	*		
[26]	*		*		MO				
[27]	*		*		MO	*	*		*
[28–33]	*		*		MO	*	*		
[34]	*		*		MILP	*	*		
[35]		*	*		MIP				
[36]		*	*		BD	*	*		
[37]		*	*		MO		*		*
[38]		*	*		DFA		*		*
[39]		*	*		BD	*	*		
[40]	*		*		Heuristic	*	*		
[41]	*		*		Heuristic	*	*		
This paper	*	*	*	*	MILP	*	*	*	*

$$\varphi(u, \theta) = (-\ln(u))^\theta \tag{5}$$

$$F(z) = 1 - e^{-\left(\frac{z}{\lambda_1}\right)^{\theta_1}} \tag{6}$$

$$G(v) = 1 - e^{-\left(\frac{v}{\lambda_2}\right)^{\theta_2}} \tag{7}$$

$$C(F(z), G(v)) = \exp\left(-\left(\ln(F(z))^\theta\right) + \left(-\ln(G(v))^\theta\right)^{1/\theta}\right) \tag{8}$$

Where, $C(u_1, \dots, u_n)$ is joint distribution function of random variable sets u_1, \dots, u_n and φ_1 to φ_n are generator functions. z and v are two sets of random variables of wind speed.

D. Generating sample values of correlated wind speeds

It is important to find an appropriate sampling method. There are different methods for obtaining sample values e.g., Marshall and Olkin [42], McNail [43] are utilized for this purpose. In this

paper, the following method is utilized that is extracted from McNail method [44].

- a. Generate a variant V with density function G_1 and Laplace transform ψ_1 .
- b. Generate X_1 with uniform PDF.
- c. Generate X_2 to X_n from nested Gumble copula $C_{d-1}\left(u_2, \dots, u_{n+1}; \psi_2^{(1)}(.; V) \dots, \psi_n^{(1)}(.; V)\right)$.
- d. Return U_1 to U_{n+1} , where $U_i = \psi_1\left(-\frac{\ln(X_i)}{V}\right)$ for $i = 1$ to $n + 1$.

In the above recursive method, U_1 to U_{n+1} are the sample values of correlated wind speeds. Ψ_i and $\Psi_i^{(1)}$ are Laplace trans-

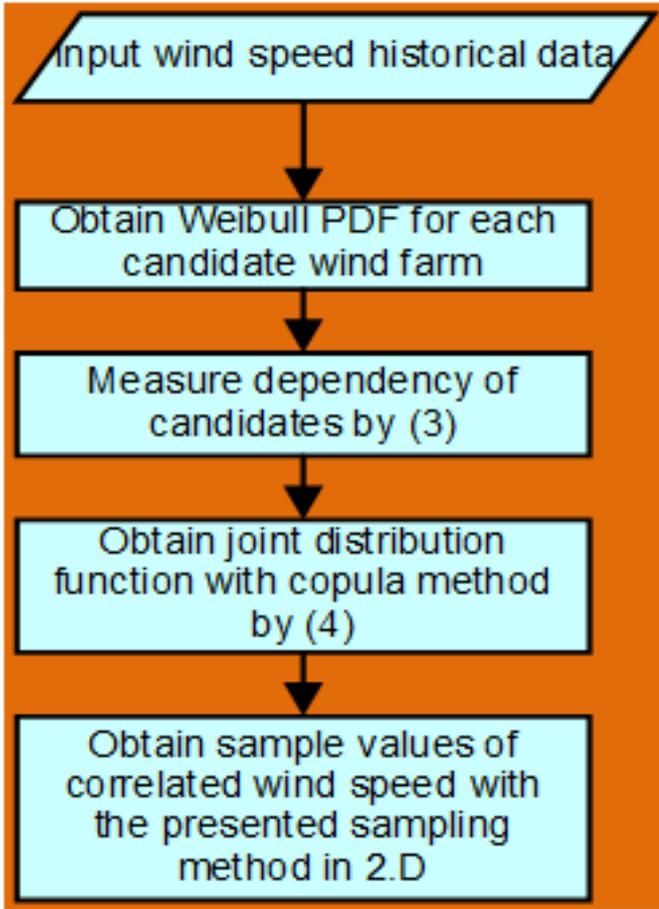


Fig. 2. Generating samples of correlated wind speeds

form and inner Laplace transform of Gumble copula, that are shown in (9) to (10), respectively. Variant V is given in (11).

$$\psi_1(t, \theta) = \exp\left(-t^{\frac{1}{\theta}}\right) \quad \theta \geq 1 \tag{9}$$

$$\psi_2^{(1)}(t; u, \alpha) = \exp\left(-ut^{\frac{1}{\alpha}}\right) \tag{10}$$

$$V = St\left(\frac{1}{\theta}, 1, \left(\cos \frac{\pi}{2\theta}\right)^\theta, \theta\right) \tag{11}$$

Where, St is positive stable distribution, θ is the dependence parameter and $\alpha \in (0, 2)$. The flowchart of generating correlated wind speed is shown in Fig. 2.

3. ULTI STATE MODEL OF WIND FARMS

Multi-state model of wind farms is considered in the researches with different methods [4-7]. In some researches, Forced outage Rate (FOR) of wind turbines is taken into account [6] and it is shown that FOR of wind turbines has a few impact on reliability analysis. Therefore, it is not considered in this paper for the sake of simplicity. The following steps are considered for multi-state modelling of wind farms.

- a. Correlated wind speeds are obtained according to section 2.
- b. Time series of the wind speeds are obtained with ARMA model according to (12).

$$ARMA(n, m) = \sum_n \sigma_n x_{t-n} + \alpha_t + \sum_m \tau_m \alpha_{t-m} \tag{12}$$

Table 2. Multi-state model of the wind farms

Output power (MW)	Data range	Probability
0	$P_w^{s_1} = 0$	$\frac{n_{s_1}}{N}$
$\frac{\sum_{s_2} P_w^{s_2}}{n_{s_2}}$	$0 < P_w^{s_2} \leq \frac{P_{wri}}{n-1}$	$\frac{n_{s_2}}{N}$
.....
$\frac{\sum_{s_{n-1}} P_w^{s_{n-1}}}{n_{s_{n-1}}}$	$\frac{(n-3) \times P_{wri}}{n-1} < P_w^{s_{n-1}} \leq \frac{(n-2) \times P_{wri}}{n-1}$	$\frac{n_{s_{n-1}}}{N}$
$\frac{\sum_{s_n} P_w^{s_n}}{n_{s_n}}$	$\frac{(n-2) \times P_{wri}}{n-1} < P_w^{s_n} \leq P_{wri}$	$\frac{n_{s_n}}{N}$

Where x_{t-n} is the previous data of wind speeds and α_t is the white noise with zero mean. The coefficients α_t and σ_n are estimated with different methods [44].

- c. Hourly wind speeds should be calculated according to (13).
- d.

$$Wh_t = \mu_t + \sigma_t y_t \tag{13}$$

Where, Wh_t is hourly wind speed, and μ_t and σ_t are mean and standard deviation of wind speed, respectively at hour t .

- e. Output power of wind farms should be calculated for each hour according to [10].
- f. Sum of the output powers is calculated and is clustered in some groups according to column 2 of Table 2.
- g. Probability of each group is obtained in column 3 of Table 2.
- h. Table 2 shows a single multi-state model for all wind farms.

Where, P_{wri} is the maximum value of sum of hourly output power that is clustered into n groups, s_1 to s_n . For example, $P_{wi}^{(s_n)}$ is a random variable in group s_n , and n_{s_1} to n_{s_n} are the numbers of random variables in the groups s_1 to s_n , respectively. N is the total number of random variables. Fig. 3 shows the steps that should be considered for preparing multi-state model of the wind farms.

4. SCENARIO CREATION

Scenarios of the outages of equipment are considered for reliability evaluation. Two scenario creation methods are given here. Monte Carlo is given in 4.1. In 4.2, a heuristic approach is taken into account that reduces the number of scenarios. In the following subsections, scenario creation methods are described.

A. Conventional Monte Carlo method for scenario creation

Monte Carlo for creating wind speed scenarios is given in the following steps [22]:

- a. $n = 1, EENS(0) = 0$.
- b. Random variables of the wind speeds are created according to Weibull PDF.
- c. A uniform PDF should be assigned to the outage of generation units and transmission lines according to FOR of them.

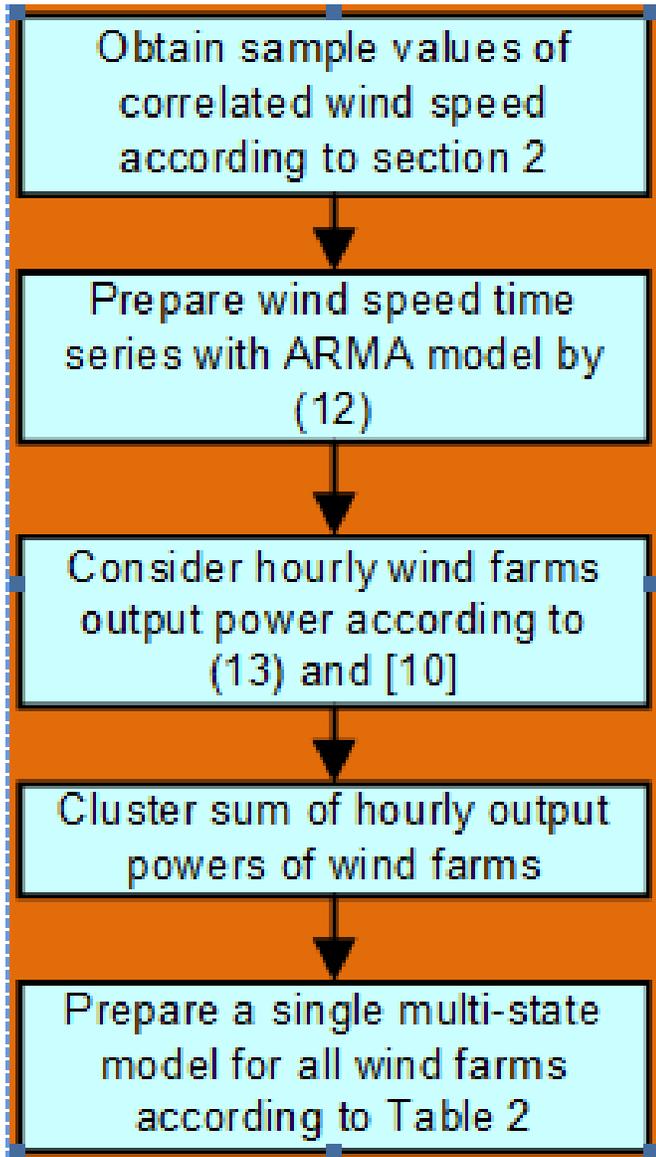


Fig. 3. Flowchart of creating multi-state model of wind farms

- d. Scenarios of wind speed are obtained by sampling from Weibull PDF and output power is calculated according to [10].
- e. Generate a number from the uniform PDF of FOR of each generation unit and line that is determined in step c, and compare it with its unavailability. If this number is less than its unavailability, then the line and generation unit is out of service, otherwise it is in service.
- f. Select scenarios with regard to (d) and (e).
- g. EENS is calculated.
- h. If $[EENS(n) - EENS(n-1)]/EENS(n) > \epsilon$, $n = n + 1$ and go to (d).

Where, n is the iteration number.

B. Heuristic method

A simple heuristic method is proposed in the following process and it reduces the number of scenarios significantly because it always converges while n is less than four.

- a. $n = 1, EENS(0) = 0$.
- b. Prepare a single multi-state model for all wind farms.
- c. Sort transmission lines according to their capacity. In this way, more effective lines on reliability indices are determined.
- d. Sort generation units according to their capacity. In this way, more effective generation units on reliability indices are determined.
- e. Consider a derated state for wind farms according to the multi-state model.
- f. Consider outages states of n transmission lines with more capacities according to step (c).
- g. Consider outages states of n generation units with more capacities according to step (d).
- h. Select scenarios with regard to (e), (f) and (g)
- i. EENS is calculated.
- j. If $[EENS(n) - EENS(n-1)]/EENS(n) > \epsilon$, $n = n + 1$ and go to (e).

Where, n is the iteration number.

In above process, reliability indices converge in less time than Monte Carlo method because:

- Multi-state model is considered for wind farms. In this way, derated states of wind farms are decreased significantly
- The outage states of units and lines that are more effective on reliability indices are considered as scenarios.

In this process, first, the outage states of one units and one lines with more capacities are considered and EENS is calculated. Then, outage states of two units and two lines with more capacities are considered and EENS is updated. This process continues until EENS does not change substantially according to step (j). In this method, outage of more effective states are considered first, thus, the process converges in less time than Monte Carlo. The above process is a heuristic method that reduces the running time of the problem and its performance will be shown in the numerical results of section 6.

5. COORDINATED TEP AND GEP

In this section, coordinated TEP and GEP is presented with a BD method. This method reduces the complexity and running time of the problem. The investment cost of new lines, thermal units, wind farms, operational cost of generation units and emission cost are minimized in the objective function that is shown in (14). The coefficient P that is shown in (15) is used for obtaining the present values of operational and emission cost. The problem formulation is decomposed into master and slave problems that are given in the following subsections. m_{ui} and n_{ij} are decision variables of unit i and line ij , respectively. If they are 1, the unit i and line ij are built and if they are 0, the unit i and line ij are not built.

$$\text{Min} \left(\begin{array}{l} \sum_{ij \in \Gamma} C_{ij} n_{ij} + \sum_{u \in UI} \sum_{i \in B} C_{ui} m_{ui} + \sum_{w \in WI} \sum_{i \in B} C_{wi} m_{wi} + \\ P \sum_{s \in S} \sum_{i \in B} \sum_{u \in UI} (C_{ui,s}^o + C_{ui,s}^e) \times m_{ui} \end{array} \right) \quad (14)$$

$$P = ((1+q)^a - 1) / (q(1+q)^a) \quad (15)$$

A. Slave problem

In BD method, the decision variables are obtained in the master problem and they are considered to be fixed in the slave problem in each iteration. Therefore, the numbers of new lines, wind farms and thermal units are assumed to be fixed in the slave problem. As mentioned in section 4, scenarios are created for uncertainty consideration and reliability evaluation in the slave problem. The index 's' shows the variables in scenario 's'. The operational cost of thermal units and the emission cost are minimized in the slave problem as shown in (16). The operational cost of thermal units consists of maintenance and fuel costs as shown in (17). Where c_{ui}^m and c_{ui}^f are the maintenance and fuel costs of thermal unit u in bus i, respectively. Fuel cost is a variable and maintenance cost is a fixed cost. Thus, the operational cost is shown with a first order linear equation in (18). The penalty costs of CO₂, NO_x and SO_x that are the main pollutant factors are included in the emission cost as shown in (19). Where c_{ui}^{NOx} , c_{ui}^{SOx} and c_{ui}^{CO2} are the penalty costs of NO_x, SO_x and CO₂ respectively for unit u in bus i. Emission cost is considered in proportion with the generation of thermal units according to (20). Equation (21) enforces power balance at each bus. Equations (22) and (23) show active power flow of the existing lines and their limits, respectively. Equations (24) and (25) show active power flow of new lines and their limits, respectively. $O_{ij,s}$ denotes the contingency variable of transmission line ij. If it is 0, the line ij is not in service in scenario s, and if it is 1, the line ij is in service in scenario s. Equations (26) to (28) determine the generation limits of existing and new thermal units and wind farms, respectively. $O_{i,s}$ denotes the contingency variable of generation unit i. If it is 0, the unit i is not in service in scenario s, and if it is 1, the unit i is in service in scenario s. The limits of buses voltage angles and loss of loads are shown in (29) and (30), respectively. Expected Energy Not Served (EENS) is obtained in (31).

$$\text{SP} = \text{Min} \left(\sum_{s \in S} \sum_{i \in B} \sum_{u \in UI} (C_{ui,s}^o + C_{ui,s}^e) \times m_{ui} \right) \quad (16)$$

$$c_{(ui,s)}^o = c_{(ui,s)}^m + c_{(ui,s)}^f \quad i \in B, u \in UI \quad (17)$$

$$c_{(ui,s)}^o = a_i \times P_{(ui,s)} + b_i \quad i \in B, u \in UI \quad (18)$$

$$c_{ui,s}^e = c_{ui,s}^{NOx} + c_{ui,s}^{SOx} + c_{ui,s}^{CO2} \quad i \in B, u \in UI \quad (19)$$

$$c_{(ui,s)}^e = c_i \times P_{(ui,s)} \quad i \in B, u \in UI \quad (20)$$

$$\sum_{u \in UI} P_{ui,s} + \sum_{w \in WI} P_{wi,s} + \sum_{ij \in \Gamma} A^T f_{ij,s} + Ls_{i,s} = P_{d_i} \quad i \in B \quad (21)$$

$$-M(1 - O_{ij,s}) < f_{ij,s}^0 - B_{ij}(\delta_{i,s} - \delta_{j,s}) < M(1 - O_{ij,s}) \quad ij \in \Gamma \quad (22)$$

$$-O_{ij,s} \bar{f}_{ij} \leq f_{ij,s}^0 \leq O_{ij,s} \bar{f}_{ij} \quad ij \in \Gamma \quad (23)$$

$$B_{ij}(\delta_{i,s} - \delta_{j,s}) < M(1 - O_{ij,s}) + M(1 - n_{ij}) \quad ij \in \Gamma \quad (24)$$

$$-n_{ij} O_{ij,s} \bar{f}_{ij} \leq f_{ij,s}^0 \leq n_{ij} O_{ij,s} \bar{f}_{ij} \quad ij \in \Gamma \quad (25)$$

$$-m_{ui} O_{ui,s} \bar{P}_{ui} \leq P_{ui,s} \leq m_{ui} O_{ui,s} \bar{P}_{ui} \quad i \in B, u \in UI \quad (26)$$

$$-O_{ui,s} \bar{P}_{ui} \leq P_{ui,s}^0 \leq O_{ui,s} \bar{P}_{ui} \quad i \in B, u \in UI \quad (27)$$

$$0 \leq P_{wi,s} \leq m_{wi} \bar{P}_{wi} \quad i \in B, u \in UI \quad (28)$$

$$-\bar{\delta} \leq \delta_{ij,s} \leq \bar{\delta} \quad ij \in \Gamma \quad (29)$$

$$0 \leq Ls_{i,s} \leq P_{d_i} \quad i \in B \quad (30)$$

$$EENS = \sum_{u \in UI} \sum_{s \in S} T_s \times Ls_{i,s} \quad (31)$$

B. Master problem

In the master problem, the investment costs of new lines, thermal units, and wind farms are minimized according to (32). A strong optimality cut is defined in (33). Desired reliability level is applied in (34). It is noted that EENS is calculated in (31) of the slave problem.

$$\text{MP} = \text{Min} \left(\sum_{ij \in \Gamma} C_{ij} n_{ij} + \sum_{u \in UI} \sum_{i \in B} C_{ui} m_{ui} + \sum_{w \in WI} \sum_{i \in B} C_{wi} m_{wi} + z \right) \quad (32)$$

$$z > \text{SP} + \sum_{ij \in \Gamma} \sum_{s \in S} \beta_{ij,s} \bar{f}_{ij} O_{ij,s} (n_{ij} - \hat{n}_{ij}) + \sum_{i \in B} \sum_{u \in UI} \sum_{s \in S} \lambda_{i,s} \bar{P}_{ui} O_{ui,s} (m_{ui} - \hat{m}_{ui}) + \sum_{i \in B} \sum_{w \in WI} \sum_{s \in S} \alpha_{i,s} \bar{P}_{wi} O_{wi,s} (m_{wi} - \hat{m}_{wi}) \quad (33)$$

$$EESN \leq \bar{EESN} \quad (34)$$

$$n_{binary}, m_{binary}, i \in B, ij \in \Gamma \quad (35)$$

Where, $\lambda_{i,s}$, $\alpha_{i,s}$ and $\beta_{ij,s}$ are the Lagrange multipliers of (25), (26) and (28), respectively and SP is the objective function of the slave problem in the previous iteration.

The flowchart of the proposed method is shown in Fig. 4.

6. RESULTS AND DISCUSSION

In this section, the proposed method is applied to IEEE 118-bus and 24-bus test systems on a laptop computer with a 2.6 GHz core i7 processor, 16 GB of RAM, and MATLAB 2015 is used. The FOR of the lines and units are assumed to be 0.02 and 0.03, respectively. The costs are given in Table 3. The specifications of IEEE 118-bus and IEEE 24-bus systems can be found in [45,46].

A. Comparing heuristic and Monte Carlo methods for scenario creation

In Table 4, the proposed scenario creation method is compared with conventional Monte Carlo for IEEE 118-bus system. It is shown that the number of scenarios and total running time are reduced significantly in the proposed method.

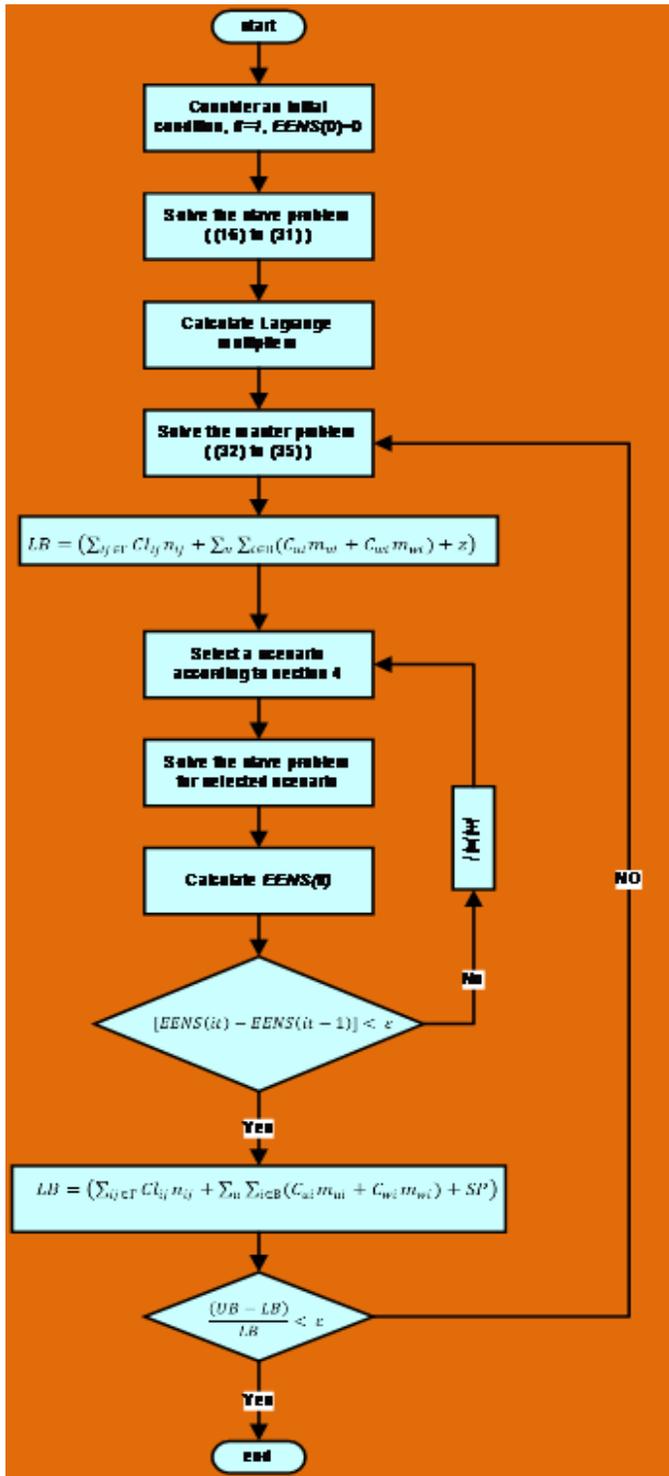


Fig. 4. Flowchart of the proposed algorithm

B. IEEE 24-bus RTS

IEEE 24-bus RTS specifications are shown in [45]. Candidate units and lines are demonstrated in Table 5 and the existing lines are also considered as candidate lines. The total load and EENS limit are 8550 MW and 7000 MWh/year, respectively. The correlation between wind farms is evaluated in this case study and the proposed copula method is taken into account for generating correlated samples of the wind speeds. Two wind

Table 3. The costs of units and lines

Item	Cost
Transmission line Investment cost	400 \$/km
Thermal unit Investment cost	1 M \$/MW
Thermal unit operational cost	20 \$/MWh
Emission cost	15 \$/MWh
Wind farm Investment cost	1.5 M \$/MW

Table 4. Comparison of the proposed method and Monte Carlo for scenario creation in IEEE 118-bus

	Proposed method	Monte Carlo
Average number of scenarios	7526	11289
Total running time	1h, 37m	3h, 19m

regimes are assumed with ARMA (4, 3) according to [6] in the buses 2 and 14. The capacity of each candidate wind farm is 2500 MW. The results are evaluated in the following sections.

a. Impact of coordination of TEP and GEP: In this section, two cases are considered:

Case 1: The problem is solved without coordination of TEP and GEP. First, GEP problem is solved and then TEP problem is considered with regard to the solution of GEP.

Case 2: The coordinated TEP and GEP problem is solved.

The results are shown in Tables 6 and 7 for two cases. It is shown that the total cost of case 1 is more than case 2, because in case 1, generation units are obtained without considering new lines. However, in case 2, generation units are determined in load centers with regard to the location and capacity of existing lines. In this way, minimum investment cost of new units and lines is obtained.

b. Impact of correlation: Correlation is an important factor in wind farms penetration. In this paper, correlation is modeled with a copula method that was given in section 2. In Fig.5, optimal wind farm integration is shown in different correlations. It can be seen that optimal wind integration is increased significantly by decrement of correlation coefficient. The impacts of correlation between wind farms on pollution cost and total cost are shown in Figs.6 and 7, respectively. It is demonstrated that the pollution and total costs are reduced effectively in lower correlations. Fig. 6 shows that lower correlation coefficients are more effective for large wind farms with high wind speed; therefore, it is recommended to consider the places with high wind speed and low correlations as candidate wind farms. As shown in Figs 5 to 7, considering correlation influences the optimal value of wind farms capacity significantly and according to the large investment of wind farms, correlation is very important factor that should be taken into account in determining wind power integration.

C. IEEE 118-bus test system

IEEE 118-bus system specifications are shown in [46]. The candidate units and lines are demonstrated in Table 8. In addition, the existing lines are also assumed as candidate lines and total load is 7220 MW. Candidate wind farms are given in Table 9. In

Table 5. Candidate units and lines in IEEE 24-bus RTS

Candidate lines		Candidate units	
To	From	Capacity (MW)	Bus no.
2	8	576	1
6	7	900	7
13	14	645	15
14	23	465	16
16	23	1200	18
19	23	1200	21
		900	22
		2500(wind)	14
		2500 (wind)	2

Table 6. New units and lines in IEEE-24 bus RTS with coordinated TEP and GEP

New units		New Lines			
Bus no.	Capacity (MW)	From	To	From	To
7	763	6	10	10	12
15	645	7	8	12	23
16	465	10	12	15	21
18	1077	12	23	19	20
22	900	20	23	1	5
14	1500 (wind)	13	14	4	9
2	1900 (wind)	6	10	3	24
		7	8	15	21
Total Cost (M\$)	20604	352			

this case, correlation is not taken into account and other factors on wind integration are evaluated in the following subsections.

a. Optimal combination of wind and thermal units: In this section, an existing wind farm is considered in bus 27 and the impact of wind farms integration in composite TEP and GEP is evaluated. Fig. 8 shows that there is an optimal point for wind farm integration in each wind speed. If wind farms penetration is more than the optimal point, total cost is increased. Therefore, there is a limit for wind farms integration even in the places with high wind speed and according to the large investment of wind farms, determining optimal wind power integration is important. The impact of wind power integration on emission cost is demonstrated in Fig. 9. It is shown that emissions are reduced

Table 7. New units and lines in IEEE-24 bus RTS without coordinated TEP and GEP

New units		New Lines			
Bus no.	Capacity (MW)	From	To	From	To
1	576	6	10	12	23
7	900	17	18	16	17
18	900	7	8	19	20
21	600	1	2	1	5
22	900	21	22	12	23
14	1700 (wind)	10	12	3	24
2	1700 (wind)	15	21	14	23
		10	12	13	14
		20	23	15	24
		21	22	1	2
		4	9	9	12
		6	10	7	8
		23	16		
Total Cost (M\$)	20500	516			

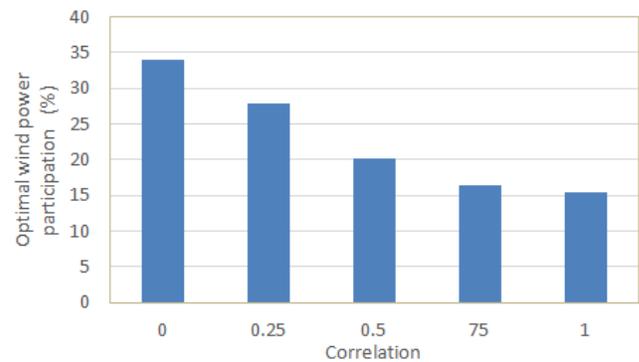


Fig. 5. Optimal wind penetration in 24-bus system

with the increment of the wind farms integration. Optimal wind farms integration is shown in Fig. 10 in different wind speeds. It can be seen that the optimal capacity of wind farm is increased with increment of the wind speed.

For determining the maximum penetration of wind farms in a network, the number of candidate place is increased according to Table 9. The impacts of increasing wind farms candidates on wind power integration and total cost are shown in Figs. 10 and 11, respectively. It can be seen that the total cost reduction tends to zero for three candidate places, thus it can be said that optimal wind power integration is limited and does not increase even with more candidate places. However, increment of candidate wind farms increases the share of wind power and it is important to investigate wind atlas of the regions to determine more candidate places of wind farms.

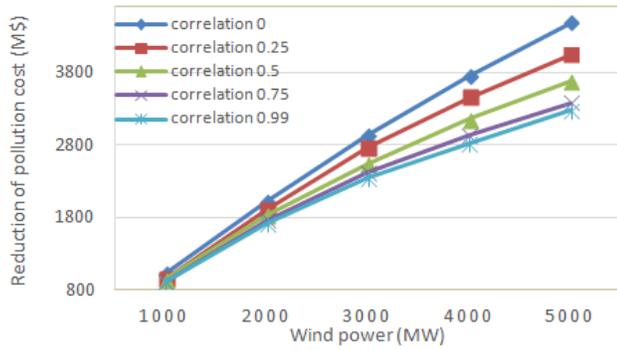


Fig. 6. Impact of correlation on pollution cost in 24-bus system

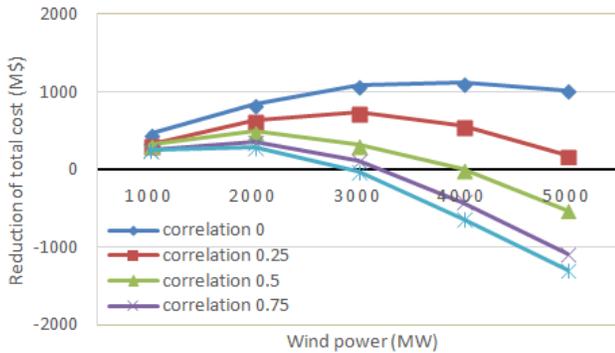


Fig. 7. Impact of correlation on total cost in 24-bus system

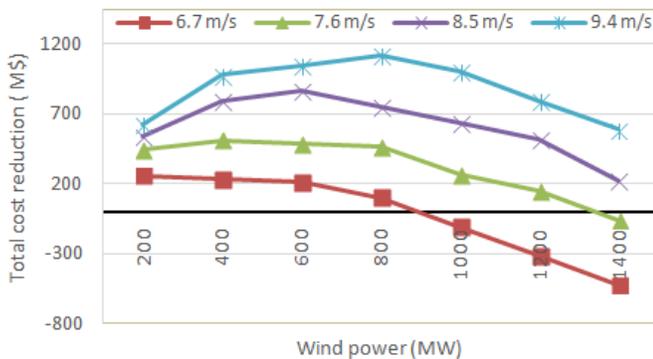


Fig. 8. Impact of different wind integration on total cost in 118-bus system

Therefore, higher wind speeds and more candidate wind farms, increase the wind power integration. However, it should be noted that there is an optimal capacity for wind power penetration that should be determined in the expansion planning.

b. Impact of EENS limit (desired reliability level): Desired reliability level constraint is considered in the master problem according to (4). Reliability level has a significant impact on wind power integration because the system reliability is maintained by combination of new wind farms and thermal units. Fig. 13 shows that total cost is increased in higher reliability levels. Therefore, reducing desired reliability reduces the total cost substantially. Optimal wind power integration is given in Fig. 12 for different reliability levels. It is shown that increasing the reliability level decreases the wind power integration. It can be

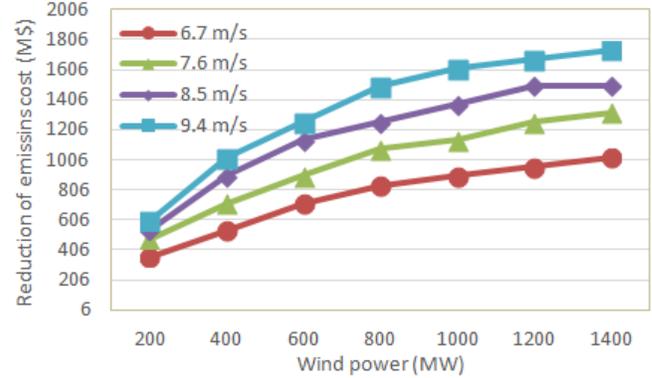


Fig. 9. Impact of different wind integration on emission cost in 118-bus system

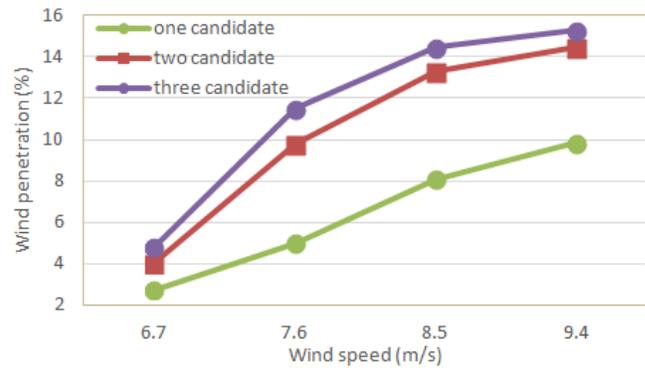


Fig. 10. Optimal wind power integration in 118-bus system

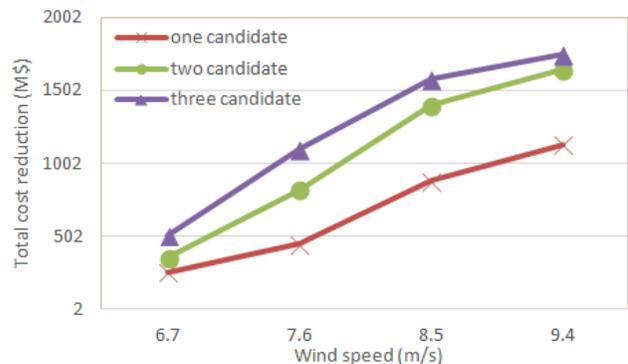


Fig. 11. Impact of optimal wind integration on total cost in 118-bus system

seen that reliability level is a substantial factor in limiting wind power penetration. The trade-off between reliability level and total cost should be considered according to available budget.

c. Impact of emission penalty: Emission penalty is considered in the objective function and it is an effective factor for determining wind farms integration. By increment of emission penalty, the total cost of thermal power is increased. Therefore, the quota of thermal power is decreased in future generation capacity and the emission is also decreased. The impacts of the emission penalty on wind and thermal power integrations are demonstrated in Figs 14a and 14b, respectively. Where, C_e is the emission cost. It can be seen that wind farms integration is

Table 8. Candidate and new lines in IEEE 118-bus

Candidate lines				New lines			
from	to	from	to	from	to	from	to
54	59	37	39	63	64	40	41
56	59	37	40	64	61	56	57
55	59	30	38	65	68	51	58
98	100	39	40	37	39	99	100
59	60	40	41	39	40	40	41
59	61	40	42	68	116	80	98
60	61	41	42	60	61	92	102
24	72	96	97	94	95	101	102
71	72	43	44	63	59	85	86
71	73	56	57	63	64	12	117
70	74	50	57	37	39	63	59
95	96	56	58	39	40		

Table 9. Candidate wind farm in IEEE 118-bus

Bus no.	Candidate capacity (MW)	Mean wind speed (m/s)	Standard deviation (m/s)
22	800	8.5	3.1
27	1500	7	2.8
50	1500	7.5	2.1

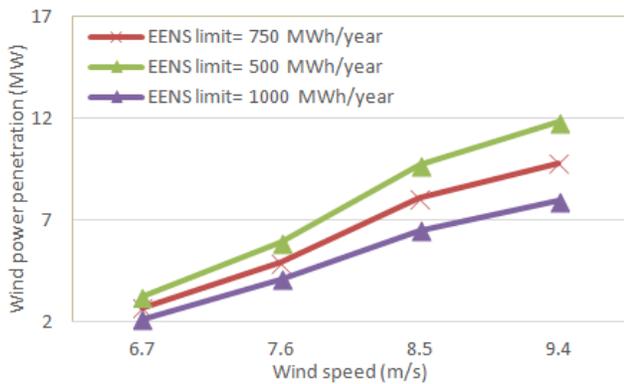


Fig. 12. Impact of EENS limit on wind power penetration in 118-bus system

increased in higher emission penalties. Fig. 14b shows that thermal power integration is reduced with increment of emission penalty.

Therefore, in the places with higher pollution level, emission penalty should be increased for obtaining more wind farms instead of thermal units. In this way, the pollution level is decreased.

Table 10. Candidate and new units in IEEE 118-bus system

No. units	Candidate capacity (MW)	New capacity (MW)
10	550	0
12	185	0
22	800 (wind)	630
25	320	0
26	414	414
31	107	0
46	119	119
49	304	0
54	148	148
59	255	0
61	260	260
65	491	350
66	492	380
69	805.2	0
80	577	0
87	104	0
89	707	0
100	352	0
103	140	0
111	136	133

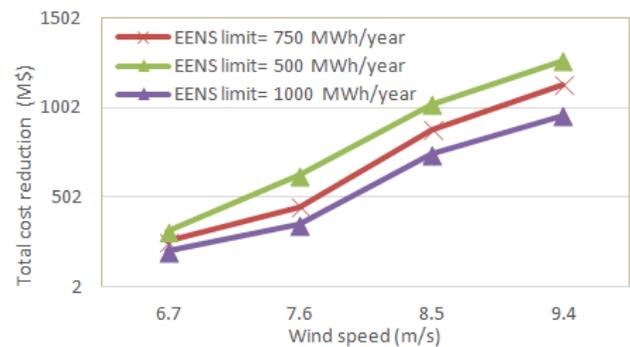


Fig. 13. Impact of EENS limit on total cost in 118-bus system

D. Comparing simulation results

For showing the performance of the presented method, the results are compared to [26]. In this reference, some kinds of units including gas, wind and hydro are considered, and two states (in or out) are assumed for the wind farms as the other units; thus, in this paper, two states for the wind farms are taken into account, and the candidate units and lines are considered the same as [26]. The results are compared in Table 11. It can be seen that in this paper, EENS and the total cost are lower than [26]. A heuristic method is used in [26]. However, a mathematical approach is utilized in this paper, and the better results are obtained.

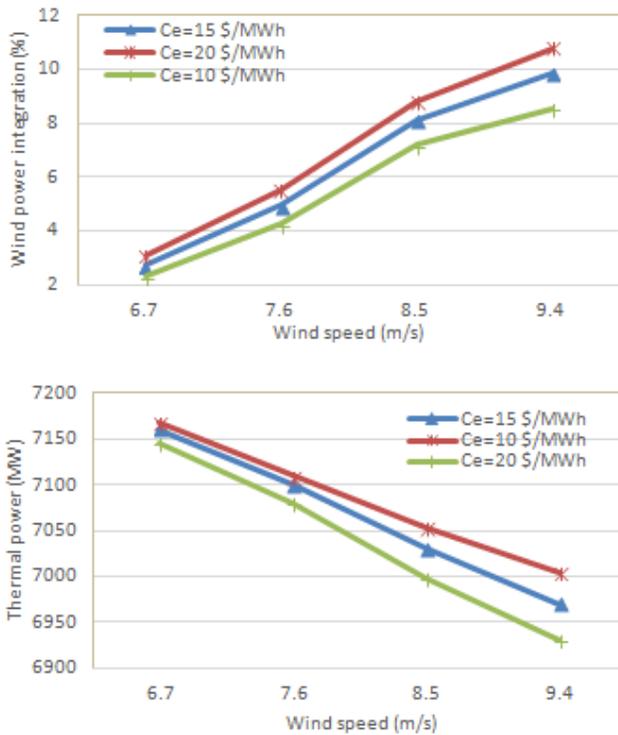


Fig. 14. Impact of emission penalty in 118-bus system a Wind power integration b Thermal power integration

Table 11. Candidate and new lines in IEEE 118-bus

Proposed method	
New lines	L.187, L.188, L.189, L.190, L.191, L.193, L.194, L.196, L.197, L.198
New units	H90.1, H91.1, H24.1, G32.1, G74.2, G90.1, G103.1, W95.1, W95.2, W102.1, W120.1
Total cost	37891 (M \$)
EENS	1371 (MWh/year)
Reference [26]	
New lines	L.187, L.189, L.190, L.191, L.194, L.195, L.196, L.197, L.198,
New units	H25.1, H119.1, G32.1, G56.1, G62.1, G74.1, G90.1, G103.1, W95.1, W102.1, W120.1, G74.2
Total cost	41919 (M \$)
EENS	1423 (MWh/year)

7. CONCLUSION

In this paper, a method for simultaneous TEP and GEP considering optimal combination of wind farms and thermal units was proposed. A multi-state model was considered for wind farms and a copula method was taken into account for correlation between wind farms. Optimal wind and thermal power integration was obtained with a comprehensive model of wind farms in the expansion planning. It was shown that there is an optimal point for wind farms integration that is increased by increment of candidate places. It was illustrated that wind speed, correlation between wind farms, desired reliability and emission penalty are the main factors in increasing optimal wind power integration. Increment of desired reliability and correlation between wind farms, decreases wind integration and increment of wind speed and emission penalty, increases wind power integration. A BD method was utilized and the number of scenarios was reduced. This method can be easily applied on large-scale practical systems and its performance was shown by implementation on the

case studies and comparing the simulation results.

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