Dynamic Optimization of Radiation Paint Cure Ovens; Studying Effect of the Search Direction and Step Size

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Paint cure oven as one of the most important instruments in production lines involves with many key parameters like curing rate and energy consumption. Radiation paint cure ovens usually have smaller amount of energy consumption besides to providing better curing conditions and as a result, attracts attentions of many manufacturers. Designing this type of ovens for curing paint of complicated geometries or thermally-sensitive materials is often a great inverse problem. Providing thermal conditions for the curing body to experience uniform cure all over its geometry without any zone of over-cured or under-cured is the most complicated part of the problem. Based upon previous works accomplished by the authors, in this study an optimization-based design method is presented in which the applied objective function is introduced based on equivalent isothermal temperature. It will be shown in this study that type and form of the objective function are the most principal issues in effectiveness and rate of the design process. Step sizes and direction vectors like other effective parameters in optimization process are studied in this article. Finally, the efficient method in designing curing ovens is employed for a typical geometry and evaluated. It will be shown that among the various considered methods, the Quasi-Newtonian method has halved the number of convergence steps and the differential step size has led to placement of more design points in the center of the cure window.

keywords: *Dynamic Optimization, Radiation Heat transfer, Paint Cure Ovens, Search direction, Step size.*

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NOMENCLATURE

A. Parameters:

b Height of the oven (m)

CT Curing Time (s)

CTL Curing Time Level (s)

C_0 Constant thermo-physical property in Eq.(1)

C_i Total heat capacity of the element i (J/K)

$E_{b,i}^k$ Emissivity power of element i at time level k (W/m^2)

EIT Equivalent Isothermal Time (s)

F_{i-j} Radiation Shape factor between elements i and j

$F(\Phi)$ objective function

$\tilde{H}$ The Hessian Matrix

L Length of the oven (m)

n Number of element on the body and the oven

NCP Nominal Cure Point

PCO Paint Cure Oven

PCW Paint Cure Window

p_i penalty term

p_r Search direction at iteration number r

$Q_{i,rad}$ Net radiation from the element i (W)

$Q_{i,g}$ Net heat transfer rate into the element i from an external thermal reservoir (W)

t_c Curing time (s)

$T_i(t)$ Temperature of element i (K)

T_r Nominal curing temperature (K)

TH Temperature History

TT Transformed Temperature (K)

v Number of design variables

B. Greek Symbols

α_r Step size

ε_i Emissivity of element i

σ Stefan–Boltzmann constant ($W/m^2.K^4$)

`$_r$ Vector of design variables at the iteration r

Φ_{target} NCP

Φ_i Equivalent isothermal time of element i

C. Subscripts

B Body

E East boundary

$i\&j$ Element number

i, r Element i in iteration number r

H Heater

N North boundary

r Counter of Optimization iteration

S South boundary

t Time

W West boundary

k Time level

1. INTRODUCTION

Curing ovens from the view-points of curing rate and amounts of energy consumption has been of great attentions in most industries; in this regard attention is being restricted to radiation cure ovens because of providing suitable curing conditions as well as consuming less amount of energy compared to other types of ovens. Designing this type of ovens for curing paint on complicated geometries or thermally-sensitive materials is often a great deal. Complications in design are usually due to providing circumstances for the curing body to experience uniform cure all over its geometry without any zone of over-cured or pre-cured area. Among radiational paint cure ovens, the continuous type is significant from standpoints of both energy and curing rate. Commonly, the widely used paint on automobile bodies consists of 5 layers: 1. Phosphate layer 2. Electro deposition layer 3. Base coat layer 4. Top coat layer, and 5. Clear-coat layer. Fig. 1 demonstrates the paint coating process and arrangement of different layers in automotive industry.

Any step of paint coating on the automobile body is proceeded by curing in the continuous radiation oven to solidify paint particles and make the layer ready to embrace next layer [1]. Continuous oven is often in form of a long tunnel of rectangular section with a rail within; to transfer the curing body along the oven in the predefined thermal condition, and to be completely cured as a result. Works of [2] and [3] can be pointed out in field of 3-dimensional simulation of ovens of this kind. The most challenging problem in designing ovens of this kind is to achieve

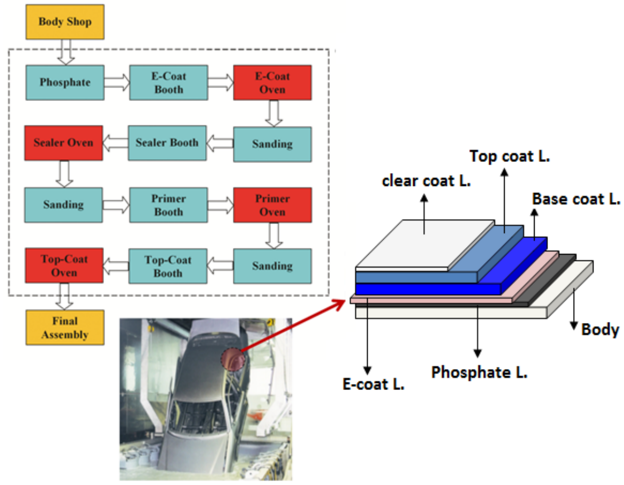

Fig. 1. Paint coating process and sequence of different layers in automotive industry.

desired cure all over the curing body by suitable arrangement of thermal sources in the oven.

Designing procedures are usually accompanied by considerable numerical costs; these costs in addition to the computation costs due to heat transfer simulations deteriorate the conditions for design problems. Therefore, proposing a fast precise approach for designing ovens is of great importance and as a result has been considered among many researchers from different points of view. Numerical simulation of the radiation heat transfer in ovens with stationary curing bodies has been discussed by many researchers. The zonal method [4], the network model [5] and the finite element method [6–9] are the most commonly used numerical methods which provide a discrete algebraic model for the problem. Simulating the heat exchanged between the radiation panels and a moving circular cylinder in a two-dimensional oven using the finite element method has been discussed by Mehdipour et al. [10].

The radiation exchange in enclosures is a classical topic discussed in many references [e.g. 11]. Computational methods applicable in the analysis of transient radiation enclosures are discussed in [12]. In most of the radiation oven applications, the assumptions of diffuse gray surfaces and non-participating medium are employed to simplify the problem.

Inverse method has widely been employed in conduction heat transfer problems. Complication of heat equations in conduction heat transfer problems usually accompanied by ill-posed solution matrices. About inverse methods works of [13–15] can be noted. Three principle sections can be defined for the optimization loop: determination of the state or field variables, evaluation of the convergence and modification of the design parameter. The latter is known as the optimizer. For the optimization stage, there are basically two classes of methods available. In a group of approaches, evolutionary methods are employed to use the objective function evaluations to update the design parameter [16, 17]. Other groups commonly apply the classical gradient-based optimization techniques [7].

Daun et al. [7, 8, 13] compared the relative merits of linear inverse and nonlinear programming methods to determine heater settings that result in prescribed conditions over a spatially fixed product surface. Yang et al. [18] used neural network model to optimize the heater settings to achieve the best possible temperature during the production process of the polyethylene terephthalate bottles. Yu [19] employed computational fluid dynamic (CFD) to simulate automotive paint curing process in an oven. He solved a well-posed problem as the heater settings in the oven were defined.

Optimum thermal design of radiation ovens with moving loads, as a sub-class of dynamic optimization problems, is computationally demanding in general. Federov et al. [20] provide a design example in which the radiation panels are thermally designed to provide a target temperature history curve on a moving flat plate. In this reference, many aspects of dynamic thermal optimization of radiation ovens are discussed in the context of a material processing example.

Dynamic optimization of paint cure problems can scarcely be found in the literature. Xiao et al. report the application of an embedded ant colony system-based optimization method to deal with this problem [16]. Zettle and Hitzmann [21] proposed an efficient method for the optimization of production parameters for baking bread rolls. The oven was predefined in their work and the robust problem of setting optimum thermal conditions was carried out during their work. Nimsuwan et al [22] simulated the temperature distribution and the flow pattern in the paint curing oven by CFD the oven as well as the heaters settings was predefined in their work. Burlon et al. [23] developed a transient model for a professional oven. They divided the whole computational domain into two thermal zones, i.e. the power zone and the cooking zone, and developed a lumped capacitance model the entire domain. Inverse methods in radiation problems have also been applied in medical treatments like radiation therapy or in tumor diagnosing [24, 25] or in image processing [26].

In this study, a new approach for designing continuous paint cure ovens is proposed. In this method, the heaters are thermally set and located to achieve a proper cure all over the curing body. Decreasing design stages has been one of the most objectives of the authors in most of their previous researches [10, 27–29] as well as the present research. In Ref. [10] a design approach based on employing cure window criterion is described and the method of defining the objective function in the design procedure is comprehensively discussed. In this study, influences of the objective function on convergence of the solution and also on the solution procedure are investigated. Before, in Ref. [27] it had been demonstrated that proper definition of the objective function decreases the solution steps while increases probability of achieving the solution. In that work, a criterion named equivalent isothermal temperature was introduced to decrease numbers of optimization steps. Decrement in numerical costs by employing the equivalent isothermal time criterion is studied in Ref. [28]. In that reference, combination of neural network and finite element method improved the approach abilities and made it capable of modelling complicated geometries with high rates. In this study, according to applying hybrid optimization method as well as some simplifications of the principle model, simulation and designing rate is increased. This design method can be applied for all geometries of any complication (this issue is investigated in Ref. [29] for a 2-dimensional geometry).

In order to solve the problem numerically it is necessary to employ some regularization methods. Importance and effects of regularization in inverse problems have been comprehensively discussed in [30]. Significance and influence of step size in optimization problems have been studied in [31].

Regarding the accomplished studies, employing optimization methods for designing continuous ovens requires proper definition of step size and search direction. This issue influences

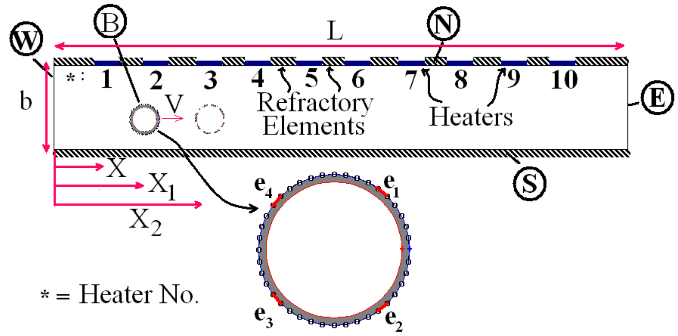

Fig. 2. A two dimensional radiation oven.

solution rates and convergence. In the present study to further develop previous works, different methods for proper definition of step size and search direction is investigated and evaluated. Some innovative methods are proposed for defining step size and performances of these methods are compared and evaluated in a sample problem of low dimensions. The approved method is then employed for designing a paint cure oven with 10 heaters.

Here we narrowly focus on the appropriate definition of the objective function in dynamic optimization of a radiation paint cure oven. With applications in auto-industry in mind, a simple circular geometry is discussed and an Arrhenius type model is used for the paint drying as suggested by Turie [8] and by use of finding the best method of search direction and step size, achieving the answer is assured.

2. PROBLEM DESCRIPTION

In the present study different methods of optimization are compared to evaluate the best approach for designing radiation ovens. In this regard a simple geometry as shown in Fig. 2 is considered. The oven geometry as well as the curing body and ten installed heaters at the oven ceiling are demonstrated in Fig. 2. The curing body enters into the oven from the left and leaves it at the right end. Designing radiation oven consists of finding the heaters temperatures and thermal arrangement of heaters in a way that, thermal conditions in the oven leads to the desired curing for the considered body.

A. Cure criterion

The temperature level and the heating duration are the two major influential factors in any baking or curing process, paint producers conduct a number of experiments in which a painted surface is cured in an isothermal environment for different durations. Based on the experimental results and theoretical considerations, a paint cure window (PCW), shown in Fig. 3, is derived that specifies the allowable curing time, CT, at different temperature levels [10]. The factory-provided PCW, therefore, defines the design domain in the temperature-curing time (T-CT) diagram shown in Fig. 3.

In continuous ovens regarding the transient thermal conditions, employing the mentioned criterion is accompanied by some difficulties. To overcome such a trouble, Xiao [32] suggested to use the parameter of equivalent isothermal time (EIT) in the curing window. The method is completely explained in [10]. In the method presented by Turrie [32] the equivalent curing time of transient condition is substituted with a reference

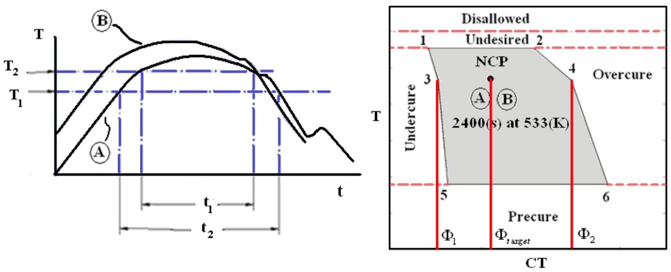

Fig. 3. TH curves for arbitrary body points A and B (a), and corresponding EITs (b).

temperature and employed in the modelling procedure. The mentioned criterion is presented as:

$$d\Phi_i = \exp[C_0(\tfrac{T_i(t)-T_r}{T_i(t)T_r})]\Delta t \qquad C_0 = \tfrac{E}{R} \tag{1}$$

$$\Phi_i = EIT_i|_{Tr} = \int_0^{tc} \exp\left[C_0(\frac{T_i(t)-T_r}{T_i(t)T_r})\right]\Delta t \tag{2}$$

where T_r is the reference temperature; The parameters E and R are paint activation energy and the gas constant, respectively. Based on this definition, if the equivalent isothermal time lies in the curing window, the desired curing is achieved. In this study evaluation of the curing is accomplished by employing the equivalent isothermal time criterion. For the critical regions of the curing body this criterion is computed and compared with the curing window criterion.

As can be observed in Fig. 3, two distinct temperature profiles can have similar equivalent isothermal time (point X in Fig. 3b). Therefore, applying the equivalent isothermal time criterion leads to considering more probable solutions [10].

According to what mentioned above, a suitable objective function can be defined in the following form:

$$\begin{aligned} &F_{avg}(\theta) = \tfrac{1}{n_B}\sum_{i=1}^{n_B} a_1\left[\Phi_i(\theta) - \Phi_{target}\right]^2 + P_i(\theta) \\ &\cdot\ P_i(\grave{}) \equiv a_2[\Phi_{\max} - \Phi_i(\theta)]^2 H\left[\Phi_{\max} - \Phi_i(\theta)\right] \\ &+a_3[\Phi_i(\theta) - \Phi_{\min}]^2 H\left[\Phi_i(\theta) - \Phi_{\min}\right] \end{aligned} \tag{3}$$

where Φ_i is the Turrie criterion for the i^{th} element while Φ_1, Φ_2 and Φ_{target} denote the curing time needed in constant temperature condition for points 4,3 and X in Fig. 2b. a_1, a_2 and a_3 are weight factors and $H(\Phi)$ is representative of step function.

After achieving temperature profile of each point as a function of time, employing equation (2) will result in determination of equivalent curing time (Φ_i). The first term becomes minimum when the equivalent curing time of the considered element equals to the time at the center of the curing window (NCP). This condition is associated with the optimum curing. The term P_i is the penalty term and acts when some points lie out of the curing window. At such conditions, this term will result in intensive increase in the objective function. The objective function, in the above-mentioned form, increases the possibility of having solution since it is not restricted to a specific temperature profile and has a well-posed matrix.

3. MODELING

A. Heat transfer analysis in the oven

Energy balance for each gray-diffuse element is written as:

$$C_i\frac{dT_i(t)}{dt} = Q_{i,g} - Q_{i,rad} \tag{4}$$

where C_i is the total heat capacity of element i, $T_i(t)$ is the temperature of element i, $Q_{i,rad}$ denotes radiation heat transfer leaving the body and results in temperature reduction. in the above correlation accounts for the energy generated in the element which is zero except for elements including heaters. Heat balance for each element in the oven is written as:

$$Q_{i,rad} = \sum_{j=1}^{N}[E_{b,i}A_i - Q_{i,rad}\frac{1-\varepsilon_i}{\varepsilon_i} - E_{b,j}A_i + Q_{j,rad}\frac{(1-\varepsilon_j)A_i}{\varepsilon_j A_j}]F_{ij} \tag{5}$$

Shape factor is calculated employing the correlation of finite elements [9]. Simulation of curing body motion has a specific effect on calculation of heat transfer. Curing body motion is descritized to specified intervals at each of which the heat transfer conditions are assumed in quasi-equilibrium condition. For instance, at k^{th} interval, the energy balance for an element after linearization is of the following form:

$$Q^k_{i,rad} - \sum_{j=1}^{N} \begin{matrix}[\sigma(T_i^{k,old})^3 T_i^k A_i^k - \sigma(T_j^{k,old})^3 T_j^k A_i^k - \\ (Q^k_{i,rad}\frac{1-\varepsilon_i}{\varepsilon_i} - Q^k_{j,rad}\frac{(1-\varepsilon_j)A_i^k}{\varepsilon_j A_j^k})]F^k_{ij}\end{matrix} = 0 \tag{6}$$

The index 'old' in the above equation represents matrix solution at the iteration before linearization. Two variants are considered unknown for each node as: 1- the element temperature and 2- the absorbed radiation heat. Writing the above equation for each node forms a set of N equations. In order to be able to solve the equation system, N other equations are needed that are generated by applying known boundary conditions or equation (4). This method is comprehensively described in [10].

B. Sensibility matrix calculation

In all of the optimization methods in this study sensibility vector or first derivation of objective function is needed:

$$\begin{aligned} &\tilde{g}_i(\vec{\theta}_r) = \left[\begin{matrix}\frac{\partial F(\Phi_i(\vec{\theta}_r))}{\partial\theta_{1,r}} & \frac{\partial F(\Phi_i(\vec{\theta}_r))}{\partial\theta_{2,r}} & \cdots & \frac{\partial F(\Phi_i(\vec{\theta}_r))}{\partial\theta_{v,r}}\end{matrix}\right]^T = \\ &\left[\begin{matrix} g_{i1}(\vec{\theta}_r) & g_{i2}(\vec{\theta}_r) & \cdots & g_{iv}(\vec{\theta}_r)\end{matrix}\right]^T \end{aligned} \tag{7}$$

where $\vec{\theta}_r$, the design parameter is heaters' temperatures. For calculation of $\tilde{g}_i(\vec{\theta}_r)$, the following equation is applied:

$$\begin{aligned} &g_{i1}(\vec{\theta}_r) = \tfrac{\partial F(\Phi_i(\vec{\theta}_r))}{\partial\theta_{1,r}} = 2a_1(\Phi_i - \Phi_{target})\tfrac{\partial\Phi_i}{\partial\theta_{1,r}} + \\ &2a_2H(\Phi_1 - \Phi_i)(\Phi_i - \Phi_1)\tfrac{\partial\Phi_i}{\partial\theta_{1,r}} \\ &+a_2\delta(\Phi_1 - \Phi_i)(\Phi_i - \Phi_1)^2 + 2a_3H(\Phi_2 - \Phi_i)(\Phi_i - \Phi_2)\tfrac{\partial\Phi_i}{\partial\theta_{1,r}} \\ &+a_3\delta(\Phi_2 - \Phi_i)(\Phi_i - \Phi_2)^2 \end{aligned} \tag{8}$$

where $\frac{\partial\Phi_i}{\partial\theta_{1,r}}$ are partial derivatives of the objective function; Due to the suggested cure window this term is computed as:

$$\frac{\partial\Phi_i}{\partial\theta_{1,r}} = \int_{t=0}^{tc}\exp\left[C_0(\frac{T_i(t)-T_r}{T_i(t)T_r})\frac{C_0}{T_i^2}(\frac{\partial T_i(t)}{\partial(\theta_{1,r})})\right]\Delta t \tag{9}$$

where $\frac{\partial T_i(t)}{\partial(\theta_{1,r})}$ is obtained through implementing some algebraic calculations on equations (4), (5) and also exerting boundary conditions for all nodes. The above equations are employed in a solution algorithm demonstrated in Fig. 4.

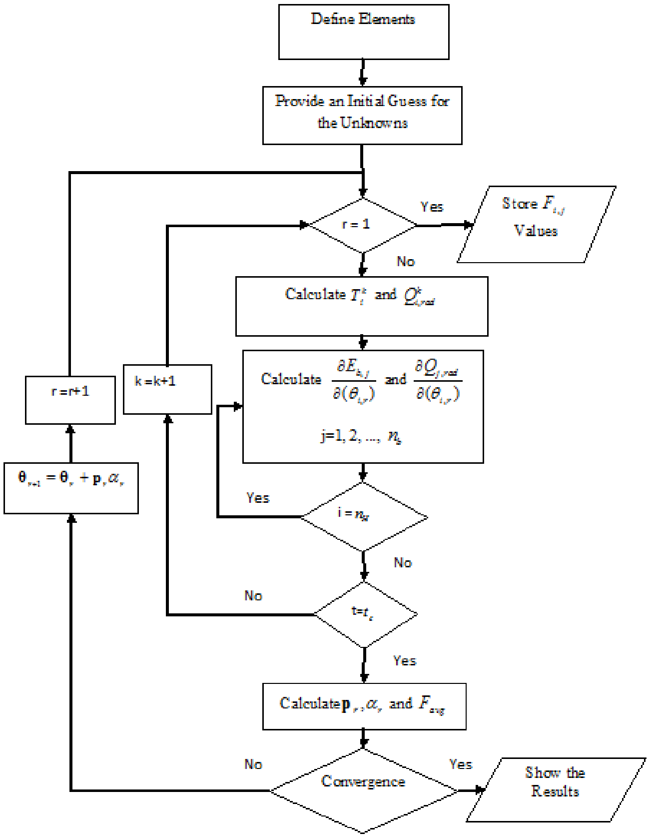

Fig. 4. The design algorithm.

4. OPTIMIZATION ALGORITHM

The employed approach for calculation of the design problem in this study is the derivation-based optimization method. In derivative approach, design parameters are corrected at each iteration, based on the following correlation:

$$\vec{\theta}_{r+1} = \vec{\theta}_r + \alpha_r \tilde{\mathrm{p}}_r \tag{10}$$

where $\tilde{\mathrm{p}}_r$ and α_r stand for search direction and step size, respectively. Computation of search direction and step size provides the conditions for modification of heaters temperatures at the next iteration of optimization.

There are various methods proposed in different references for calculation of search direction. Some of these methods are:

1 steepest-descent

2 Fletcher-Reeves conjugate gradient method (FR-CGM)

3 Polak-Ribiere conjugate gradient method (PR-CGM)

4 Stiefel-Hestens conjugate gradient method (SH-CGM)

5 First order -Quasi-Newton

6 Davidon Fletcher Powell Quasi-Newton (-DFP- QN)

7 Broyden Fletcher Goldfarb Shanno Quasi-Newton (BFGS - QN)

8 Marquardt method

For instance, the BFGS-implementation of the quasi-Newton method calculates the search direction according to

$$\vec{P}_r = -\tilde{H}_r^{-1} \vec{g}_r \tag{11}$$

where the Hessian is approximated by:

$$\begin{aligned}
&\tilde{H}_r = \tilde{H}_{r-1} + \tilde{M}_{r-1} + \tilde{N}_{r-1} \quad r = 1, 2, ... \\
&\tilde{H}_0 = I \\
&\tilde{M}_{r-1} = \left(\frac{1 + y_{r-1}^T \tilde{H}_{r-1} y_{r-1}}{y_{r-1}^T P_{r-1}} \right) \frac{\vec{P}_{r-1} \vec{P}_{r-1}^T}{\vec{P}_{r-1}^T \vec{y}_{r-1}^T} \\
&N_{r-1} = -\frac{\vec{P}_{r-1} \vec{y}_{r-1}^T \tilde{H}_{r-1} + \tilde{H}_{r-1} \vec{y}_{r-1} \vec{P}_{r-1}}{\vec{P}_{r-1} \vec{y}_{r-1}^T} \\
&\vec{y}_{r-1} = \vec{g}_r - \vec{g}_{r-1}
\end{aligned} \tag{12}$$

The elements of the gradient vector correspond to the objective function sensitivities with respect to the design parameters (heater panel temperatures):

$$\begin{aligned}
&g_i(\vec{\theta}_r) = \left[\begin{array}{cccc} \frac{\partial f_i(\vec{\theta}_r)}{\partial \theta_{1,r}} & \frac{\partial f_i(\vec{\theta}_r)}{\partial \theta_{2,r}} & ... & \frac{\partial f_i(\vec{\theta}_r)}{\partial \theta_{v,r}} \end{array} \right] \\
&= [g_{i1}(\vec{\theta}_r) \;\; g_i(\vec{\theta}_r) \;\; ... \;\; g_i(\vec{\theta}_r)]
\end{aligned} \tag{13}$$

In the process of obtaining the sensitivity vector, depending on the definition of the objective function, the magnitude of $\partial T_{i(t)} / \partial(\theta_{1,r})$ should be calculated.

After finding the sensitivity matrix, $g_i(\vec{\theta}_r)$, is obtained. The Hessian matrix, step size and vector of variations are calculated applying equations (13), (11), and (12), respectively. Then Equ. (10) can be applied to compute the heaters temperature for achieving better curing. The optimization procedure will be continued until reaching a minimum value of the objective function which is equal to approaching in to the center of cure window.

The design procedure based on optimization method, can be divided into 3 major stages. First stage is basically a thermal analysis routine in which the field or state variables are updated. At the second stage, the objective function is simply evaluated and checked against a convergence criterion. Finally, at the third stage, the design variables are updated to ultimately nullify the objective function, up to a convergence criterion. Fig. 4 demonstrates calculation procedure and optimization.

5. SIGNIFICANCE OF STEP SIZE ON THE DESIGN PROCEDURE

In different gradient methods, a function is predicted to be curve-fitted on the objective function at each step of the optimization procedure; this function is named as minimizer (Fig. 5). Since the curve-fitted function (i.e. minimizer) is usually of simple forms, its minimum point can be found easily and be a good instruction to the minimum point of the objective function. At the next step (if any) again the previously employed function is curve-fitted on the objective function at the new instructed point and the procedure goes on. These methods are more efficient in the points that the objective function is of similar order to the minimizer function. For instance, in the steepest descent method, the first derivation of objective function due to design parameter is employed. This issue is shown in Fig. 5.a. If the objective function in a 2-variant problem is of $x_1^2 + x_2^2$ form, direction and the magnitude of search direction can be computed exactly and the minimum point can be calculated in a single step. However, if the picked-out objective function is of different form to what pointed above, diminishing the magnitude of the objective function is only assured. Zigzag procedure is usually observed in these cases. The mentioned condition is demonstrated in Fig. 5(a) for an elliptical second order function.

In Newtonian method as well as quasi-Newtonian, first and second derivation of the objective function due to design parameter is applied and a function of second order is curve-fitted on the current points (Fig. 5.a). It is expected that this method because of using higher order derivatives of a point, is more efficient than the steepest descent rate method. In Newtonian method the second derivation of the objective function due to design parameter (Hessian matrix) is applied that results in high numerical costs and in some cases impossible conditions for calculation of Hessian Matrix. This problem is fixed in quasi-Newtonian method since the Hessian Matrix is guessed from the first derivative of the objective function and therefore amount of calculations in this method is similar to lower order methods. Less numerical costs along with approximately same precision to Newtonian method has made the quasi-Newtonian method widely adopted among researchers.

It should be noted that because of using more approximations, Quasi-Newtonian method is considerably sensible to the starting point of optimization as well as the style of variation of objective function. This sensitivity urges the first guess to be near enough to the minimum point to assure the convergence in solution [33].

Some criterions for evaluation of quasi-Newtonian method are eigenvalues and condition number of Hessian matrix. A negative eigenvalue points out a saddle point, since it does not simulate the condition for minimum nor maximum point. Calculating the search direction for such points in Newtonian methods are often encountered with a problem. In quasi-Newtonian methods, condition number of Hessian matrix denotes the deviation of the objective function from treatment of a second-order function.

When the first guess is far from the minimum point, the above-mentioned methods are not efficient enough and use of steepest descent is more recommended. Then as the optimizing procedure approaches to the minimum point, the Newtonian method performs better. Combination of these two methods results in a new method named Marquardt Method, with further capabilities in comparison with each method individually [34].

It should be noted that for all functions except those of second order type, the performance of optimization methods decreases [34]. In such cases, to improve efficiency of optimization procedure, it is recommended to apply the step size to modify the magnitude of search direction. When the objective function follows the second order form, there is no need to use the step size, otherwise applying the step size is essential and significantly influences convergence rate and procedure [34]. There are 5 types of step sizes explained in the following:

A. Descent step size

This type of step size is calculated using the following correlation:

$$\alpha_r = \alpha_0 / r^a \tag{14}$$

B. Cross step size

Cross step size is calculated by the following relation:

$$\alpha_r = (\frac{\alpha_0}{r^a})(\frac{1}{(\frac{r}{c})^{(br-2)}} + 1) \tag{15}$$

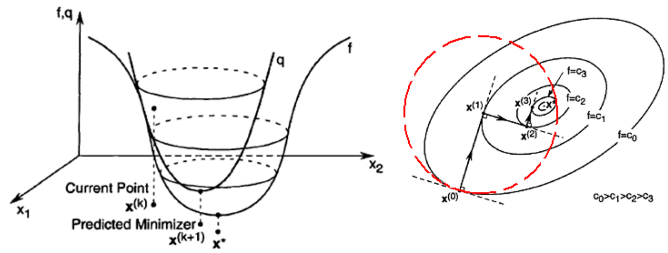

Fig. 5. Employing a) gradient method, b) steepest descent method in an elliptical form problem

C. Relative step size

This kind of step size is defined as:

$$\alpha_{r+1} = \frac{\frac{1}{v}\sum_{l=1}^{v}(\frac{I_{l,r+1}}{I_{l,r+1}-I_{l,r}})}{-\sum_{l=1}^{v}(I_{l,r+1})^2}(F(\vec{\theta}_{r+1}) - F(\vec{\theta}_r))$$
$$I_{l,r} = (g_l(\vec{\theta}))_r = (\frac{\partial F}{\partial \theta_l})_r \tag{16}$$

As can be found, definition of above constants doesn't concern the problem type nor the designer experience. It should be noted that the above relation is obtained based on steepest descent method. The step size computed from equation (16) varies for different optimization methods.

D. Vectorial relative step size

In the previous method the step size was a scalar quantity. Design parameters were obtained by achieving the best step size in direction of search direction. Vectorial relative step size α_r, is defined in vectorial form introduced in equation (17):

$$\theta_{l,r+1} = \theta_{l,r} + p_{l,r}\alpha_{l,r} \tag{17}$$

Present approach for finding the step size is similar to the previous method with a single distinction; in this method, each terms of step size denotes the modified value for each of design parameters. The present step size is defined in the following form:

$$\alpha_{l,r+1} = \frac{\frac{1}{v}\sum_{l=1}^{v}(\frac{I_{l,r+1}}{I_{l,r+1}-I_{l,r}})}{-(I_{l,r+1})^2}(F(\vec{\theta}_{r+1}) - F(\vec{\theta}_r)) \tag{18}$$

E. Differential-based step size

In this method the desired value of step size is calculated by differentiating the objective function via the step size.

$$\frac{\partial f(\vec{\theta}_r + \vec{p}_r\alpha_r)}{\partial \alpha_r} = 0 \tag{19}$$

Note that the second to forth methods are the innovative approaches proposed in this study.

6. OBJECTIVE FUNCTION TREATMENT

It has been found in the solution procedure that in order to obtain characteristics of employed heaters individually, different methods of optimization without a proper step size may not achieve in desirable convergence. Some of optimization methods were not efficient in defining heaters characteristics. In this part behavior of the objective function against alteration of design parameter is evaluated. In this regard preference of some

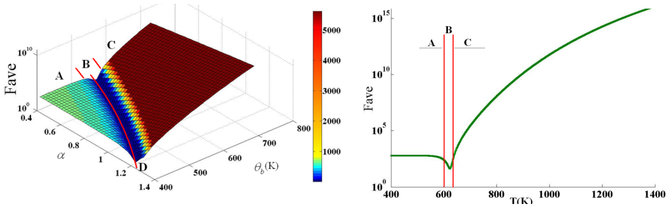

Fig. 6. Variation of the objective function against design variables.

optimization methods in comparison with others can be found (These methods are different in methods of calculating search direction/step size).

In the present problem ten parameters are involved as design parameters (ten heaters have been applied). However, variation of the objective function against the design parameter can be estimated as a ten-dimensional surface. This issue hardens demonstration of the objective function due to design parameters. In such cases behavior of the objective function is demonstrated using one of the following approaches:

1 A pre-set for heaters is assumed and a single variant (temperature of a specified heater) may be changed to study the variations of the objective function against the mentioned heater. This approach is same as studying the sensibility matrix.

2 In a pre-set, heaters temperatures are assumed to be $\vec{\theta}_c$ and then temperatures of heaters changes about a specified direction "$\vec{\theta}_b$". It is obvious that the minimum of the objective function lays in a range from under-cured to over-cured conditions. Variations of the design parameter satisfies the following relation, to reveal the objective function treatment at each step.

$$\vec{\theta} = \vec{\theta}_c + \vec{\theta}_b \qquad 400 \leq \left|\vec{\theta}_b\right| \leq 1400 \tag{20}$$

3 Two arrangements for heaters temperatures defined as $(\vec{\theta}_1, \vec{\theta}_2)$ are assumed. Applying different values for in equation (21) result in finding other possible arrangements of heaters between the two specified. Shifting in thermal levels of the obtained arrangements is then possible with choosing any value for θ_b. In this case two-dimensional region (because of appointing two arrangements) with more possible arrangements can be studied. This method applies some diagrams of second method with different primary settings together, to provide a two-dimensional curve. Heaters temperature in this approach is calculated employing following relation:

$$\vec{\theta} = \alpha\theta_1 + (1-\alpha)\vec{\theta}_2 + \vec{\theta}_b \mathrm{J} \tag{21}$$

Treatment of the objective function is demonstrated in Fig. 6 using second and third method, where J is representative of a single column identity matrix.

4 Based on this method the principal problem changes in to a problem with two design parameters. Computational domain can be completely displayed in this case. Implementation of this approach on the present problem is of following form: temperature of first five heaters varies jointly while temperatures of other heaters change together. This issue provides the condition to study behavior of the objective function in lower dimensions. Fig. 7 demonstrates this approach. The two-variant technique is more appropriate for evaluating different optimization methods as in this technique the computational domain can completely be shown. The procedure to achieve the minimum point can be studied in detail. First the whole computational domain is discretized. Every point in this domain representing a heater arrangement is modelled and the magnitude of the objective function is calculated. Finally, variation of the objective function in the computational domain is achieved (Fig. 7). The X axis denotes temperatures of first five heaters while the Y axis is demonstrative of the rest five heaters. Fig. 7(a) shows that line A is the local minimum of the computational domain and intensive alteration around this line is noticed. Little variation on this line with intensive variations in other zones hardens finding the minimum.

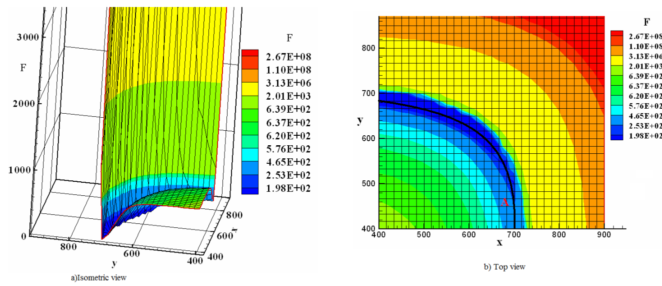

Fig. 7. Variations of the objective function based on a similar problem with two design parameters.

Although the above-mentioned approaches do not reveal the behavior of objective function generally, they are informative enough to apply. It can be observed from Fig. 6 that the objective function variations against $\vec{\theta}_b$ does not follow a second order function treatment but its variation can be mentioned in 3 different zones demonstrated in Fig. 6. As observed in the Fig., there is no remarkable variation in zone A; therefore, the objective function does not notably take affect from alteration in heaters temperatures. Contrary to zone A, there is zone C with intensive variations of objective function relative to heaters temperatures (note that the diagram is in logarithmic form). The minimum point of this curve for the present problem lies in zone B. Behavior of the objective function from left to right of Fig. 6 can be described as:

1. Small slope region (zone A) 2. Intensive slope zone (region between zone A and B) 3. Minimum point (at the center of zone B) 4. Intensive slope (Zone C and the region between C and B)

This issue becomes more evident since the search directions at the middle of zones A and C ignoring step size obtained about and respectively (obtained by steepest descent method). Existence of local minimum on line A is shown in Fig. 8. It is obvious that applying step size as the most key parameters in optimization performance is essential.

Following are some reasons for employing an efficient step size:

1 Deviation of this function from behavior of second order function

2 Having the maximum slope in region B and C.

3 Existence of some local minimums on line D.

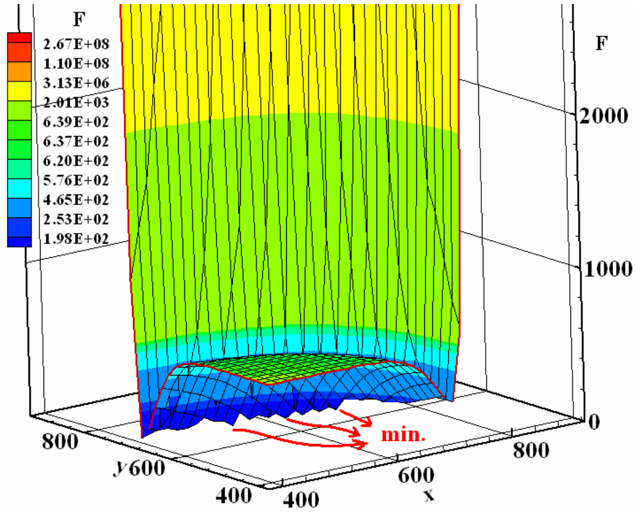

Fig. 8. Local minimums in the solution domain.

7. EVALUATION OF DIFFERENT METHODS OF DESIGN

At this part of the study, efficiency of the design method is evaluated. In order to evaluate different methods of design, a reference problem described in Fig. 1 is considered [10]. Cure window criterion employed in ref. [10] along with equivalent curing time criterion [32] are applied to assure that the problem has a solution. It is expected that a proper method, independent of what initial guesses are, can obtain heaters temperatures in a way that all target points situated in the cure window far from its boundaries.

Before studying the principle problem, it is necessary to propose methods of defining step size and search direction. Since studying this issue is much simpler in 2 dimensions, all methods are investigated in fourth methods of section 7.

Magnitude of the objective function for each point in the two-dimensional solution domain is evaluated as an illustrative of curing extent. The solution domain is demonstrated in Fig. 7. In order to find minimum point of the objective function representing optimum heaters arrangement, different gradient methods have been implemented on the present problem and their performance has been weighed up. Two different initial conditions of uniform temperature of 400K and 900K for each heater are considered. To assess performance of different methods mentioned for search directions, each method is implemented on the model and at last results are analyzed. (In this section step size is computed applying gradient method). Each solution is coded and tabulated in Table 1.

Fig. 9 demonstrates the variation procedure in various gradient methods. Lots of data along with intensive fluctuations causes some ambiguity in this Figure. Fig. 10 demonstrates performance of two methods of optimization individually.

As can be observed in Fig. 9, variations at the first step move toward the minimum line of A (Fig. 7). This motion follows the path B for the initial conditions of 900K and path D for initial conditions of 400K as shown in Fig.9. After approaching line, A, all methods move toward zone E. it should be noted that because of smooth variations around the point (400,400) during the motion in path D search direction toward the final minimum is defined properly and directly converges to zone E. Probable

Table 1. Defining characteristic codes for the proposed methods

Code	Methods of defining search direction	Methods of defining step size
K111	Steepest descent rate	Descending step size
K112	Steepest descent rate	Hybrid step size
K113	Steepest descent rate	Relative step size
K114	Steepest descent rate	Vectorial relative step size
K115	Steepest descent rate	Differential step size
K315	FR-CGM	Differential step size
K325	PR-CGM	Differential step size
K335	SH-CGM	Differential step size
K415	Quasi-Newtonian (first order correction)	Differential step size
K425	DFP- QN	Differential step size
K435	BFGS – QN	Differential step size
K433	BFGS – QN	Relative step size

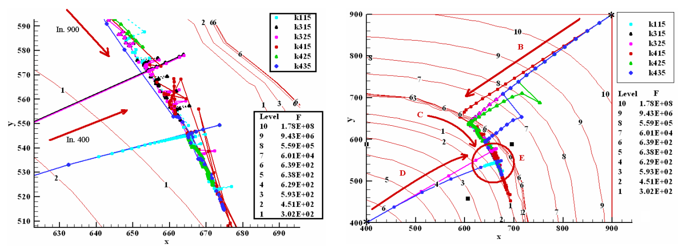

Fig. 9. Performance of different gradient methods in the solution domain.

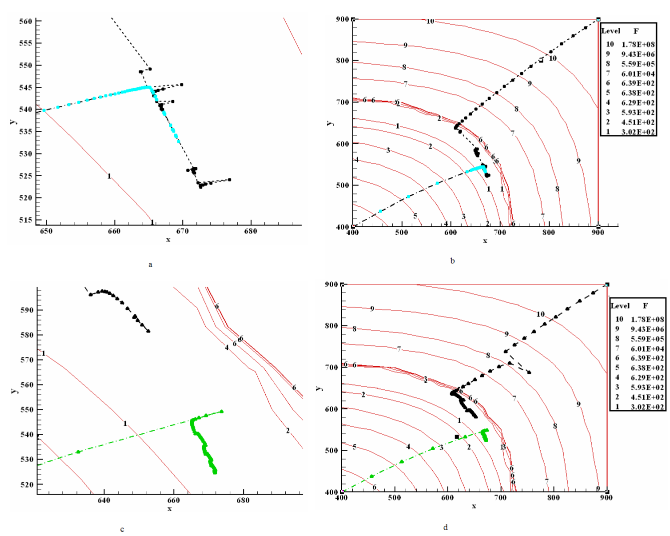

Fig. 10. a) Magnification of minimum point, b) Performance of steepest descent method (K115), c) Magnification of minimum point, d) Performance of DFP QN (K425).

convergence to different solution can be related to existence of local minimums shown in Fig. 8.

Heaters arrangement and equivalent isothermal time for each of four demonstrated points in Fig. 2 implementing all methods of design are shown in Fig. 11a and Fig. 11b, respectively. Variation of the objective function at each iteration is displayed in Fig. 12. It can be concluded that there is no significant distinction

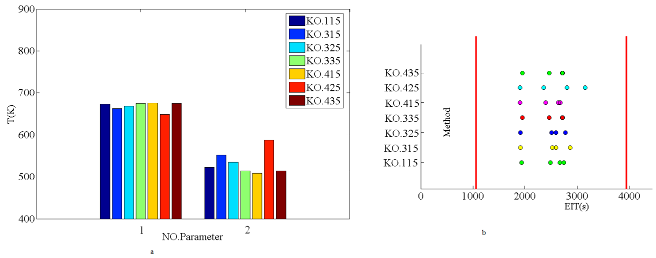

Fig. 11. a) Magnitude of design parameter (heaters temperature) at the end of design procedure b) Situation of chosen equivalent time due to the cure window.

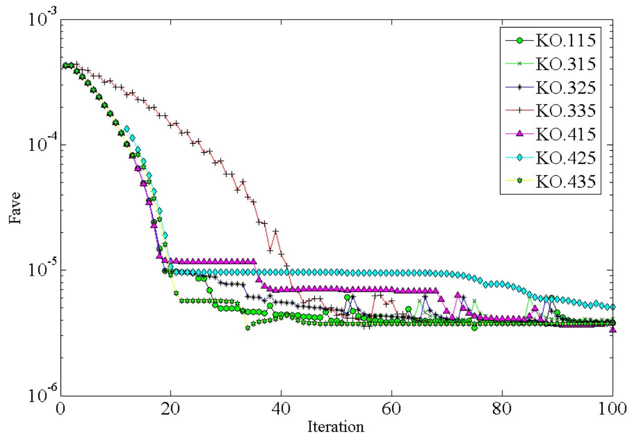

Fig. 12. Variations of magnitude of objective function against each step of optimization for different gradient methods.

between the mentioned methods. This fact reveals that proper definition of step size is more important than search direction.

Various methods are compared from the viewpoints of some criteria like: simple implementation, convergence adjacent to minimum point and convergence far from the minimum point.

Conclusion remarks as well as comparison between different methods for defining search direction are explained in Table 2. The second order quasi-Newtonian method (BFGS) together with differential step size brings about the best convergence procedure. The quasi-Newtonian method has halved the number of convergence steps in the problem.

Methods for determination of step size were studied above and a brief comparison for these methods is listed in Table. 3.

Table 2. Defining characteristic codes for the proposed methods

From the view -point of simplification of use	From the view-point of convergence rate far from the minimum point		From the view-point of convergence rate adjacent to minimum point	prominence
	At the end of optimization procedure	At the first of optimization procedure		
K115	K415, K115, K325, K315	K435, K315	K435, K325, K315	1
K325, K315	K433	K115, K325	K433,	2
K435, K433, K415	K435,	K433	K115	3
		K415	K415	4

Table 3. Comparison between performances of different methods for calculation of step size in the current problem

Prominence	From the view-point of simple implementation	From the view-point of convergence far from the minimum point	From the view-point of convergence adjacent to minimum point
1	Differential-based step size	Descent step size and Cross step size	Differential-based step size and relative step size
2	Relative step size and Vectorial relative step size	Differential-based step size	Vectorial relative step size
3	Cross step size	Relative step size and vectorial relative step size	Descent step size and Cross step size
4	Descent step size		

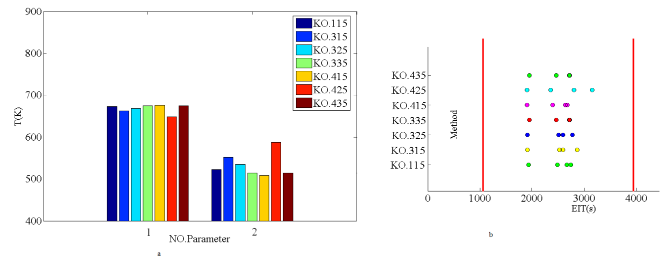

Fig. 13. Cure window criterion at the end of design procedure for different methods of step size.

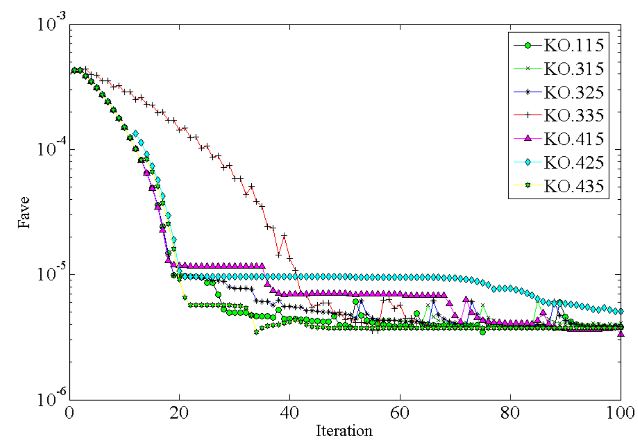

Fig. 14. Heaters temperature at the end of design applying differential optimization methods.

In this part search direction has been achieved employing the method of steepest descent rate. Cure criterion for the demonstrated elements of Fig. 2 are shown in Fig. 13. All points are well compatible with the criterion as can be observed in Fig. 13. As can be found from the figure, the differential step size has led to placement of more design points in the center of the cooking window. The criterion of the defined objective function improves from 10^{-5} to 10^{-6}.

Based on evaluation of methods available in Table 3, it has been revealed that the second order Quasi-Newtonian method (BFGS) applying differential step size has shown the most appropriate performance among other approaches. Number of heaters as well as their temperature variations, are factors representative of curing condition. Although by increasing number of individual heaters numerical costs enlarges, possibilities become available for better curing consequence. The above mentioned approach has been implemented on the geometry of Fig. 2 with 10 individual heaters and the designed heaters arrangement is demonstrated in Fig. 14.

8. CONCLUDING REMARKS

In this paper, the procedure for designing heaters in a continuous radiation oven is assessed. Effects of the step size and the search direction in optimization procedure are investigated. Some different approaches are presented for determination of step size and direction vector; at last performances of these methods together with optimization procedure are evaluated.

Influences of 5 different methods for computation of step size on the solution procedure is evaluated in this study and the most efficient method from the view-point of solution is the Quasi-Newtonian method (BFGS) with differential step size. Among the considered methods, two of them are new and claimed to be an innovation for the present work. It has been revealed in this paper that proper definition of step size prominently influences the solution procedure.

The "Quasi-Newtonian method (BFGS) applying differential step size", as the most appropriate method among all other considered methods is employed at last to design a 10-heater cure oven. Then maximum number of heaters that can be implemented on the present computational model is discussed.

The Quasi-Newtonian method has halved the number of convergence steps. It is found that the differential step size has led to placement of more design points in the center of the cooking window. The criterion of the defined objective function was improved from 10^{-5} to 10^{-6}.

Evaluation of optimization procedure and design in this paper has provided the background for making the considered methods applicable for complicated geometries with high amounts of computations.

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