Energy Hub is an appropriate framework for modeling and optimal scheduling of multi-energy systems (MES). Energy hub provides the possibility of integrated management of various inputs, converters, storage systems, and outputs of multiple energy carrier systems. However, the optimal management problem in the energy hub is affected by various technical, economic, social and environmental parameters. Many of these parameters are inherently ambiguous and uncertain. Fluctuating nature of renewable energy sources (RES), energy prices in competitive and deregulated markets, the behavior of consumers, inherent variations in the surrounding environment, simplifications and approximations in modeling, linguistic terms of experts, etc. are just a few examples of uncertainties in the optimal management problem of energy hub. Ignoring such uncertainties in the process of modeling and optimization of energy hub leads to unrealistic models and inaccurate results. On the other hand adding these uncertainties leads to increased complexity of modeling and optimization. Therefore, to achieve a realistic model of MES in the form of energy hubs, identifying appropriate methods to address these uncertainties is essential. This paper reviews the different methods for the consideration of uncertainty in optimal scheduling of energy hubs. In this paper, different methods of modeling and optimization of energy hub are reviewed and classified and their strengths and weaknesses are discussed. A classification and review of the various methods that offered in the most recent research of MES in the field of uncertainty modeling are done to identify efficient methods for using in energy hub models. © 2017 Journal of Energy Management and Technology

1. INTRODUCTION

Nowadays, economic development, population growth, industrialization and increasing demand have caused energy to become one of the main components of the current human development. The hierarchical structure of generation, transmission, and distribution of energy has been the main framework for demand and supply of energy in recent decades. However, the development of distributed energy resources (DER), particularly RES and local storage systems, leads to non-hierarchical and distributed structures. On the other hand, the advent of efficient technologies, such as combined heat and power (CHP) production, electric heat pumps and fuel cells, leads to the integration of energy infrastructure, such as electricity, natural gas, and district heating networks which create MES [1]. In a multi-energy system, different energy carriers can interact optimally. The existence of various energy carriers and energy infrastructures leads to the complexity of the structure of these systems. On the other hand, optimal performance of such systems requires a comprehensive modeling framework and integrated management.

Energy hub concept has been developed in recent years for integrated management of MES [2]. Planning and management of energy hubs as an effective and efficient method in modeling and utilization of energy systems are essential for ensuring energy security, environmental sustainability, and economic development. However, the energy systems are affected by various technical, economic, social and environmental parameters characterized by uncertainty and high volatility. Therefore, optimal management of energy systems, especially MES, needs to take account the uncertainties in modeling and optimization of the systems. Increased share of renewable resources, deregulation of energy markets and behavior of the different components of the energy system such as producers and consumers, create uncertainty and difficulty in scheduling these systems. Due to
the high cost of the investment in energy infrastructure, the actual planning of these systems is crucial and taking into account the associated uncertainties to reach a real (or close to reality) decision is essential. On the other hand, energy hub operational optimization problem results are under the influence of model input parameters variations and this increase the importance and necessity of uncertainties studying in the process of managing and scheduling of these systems.

This paper reviews the various techniques for taking into account the uncertainties in the planning process and optimal management of the energy hubs. First, an introduction to the concept of energy hub is provided and various methods and models to optimize energy hub are classified and evaluated. The various techniques used in the literature for the consideration of uncertainties in modeling and optimization of energy hub are discussed. As well as, different uncertainty modeling methods that have been used in most recent MES research are reviewed and classified and the advantages and disadvantages of each of them are discussed to determine appropriate techniques for addressing uncertainty in the optimal scheduling of energy hub.

The remainder of this paper is organized as follows. Section 2 provides an introduction to the concept of energy hub and various methods for modeling and optimization of energy hub. Section 3 reviews the different methods used in the literature for modeling uncertainty in energy hub, and efficient methods have been employed in the other MES models. Section 4 includes discussions and general comments about modeling uncertainties in energy hub. The paper is concluded in section 5.

2. ENERGY HUB CONCEPT AND OPTIMIZATION METHODS

Energy hub concept was developed for the first time in “A Vision of Future Energy Networks” (VOFEN) [3] project and aims to get a vision and model of future energy systems. Moving toward MES, non-hierarchical structures, and integrated energy systems are the main goals of this project. Therefore, in order to achieve such structures of energy systems a concept called “Energy Hub” has been introduced as a unit that provides the features of input, output, conversion and storage of multiple energy carriers. Energy hub can be defined as the place where the input parameters variations and this increase the importance and necessity of uncertainties studying in the process of managing and scheduling of these systems.

The matrix model of energy hub concept is used to link different objectives corresponding to energy hub input and output through coupling matrix. Each of the matrix elements represents energy hub interior features include connections, transform coefficients and operating function of various internal components of the energy hub. This matrix provides the possibility of modeling inputs, conversion, storage systems and various outputs in a comprehensive model. Therefore, integrated management and optimization of the MES are provided in the form of energy hub model.

Optimization models related to energy hub can be categorized in different ways. For example, two basic optimization modes can be considered for energy hub as follows:

- Structural optimization: finding the optimal topology and structure of energy hub, based on a specific demand and corresponding objective functions
- Operational optimization: Optimal power dispatch in an...
energy hub or optimal power flow in the network of inter-
connected energy hubs for a given structure of the system.

Such concepts for energy hub were presented and formulated
by Geidl and Andersson. They have offered a model for opti-
mal power dispatch problem in systems with multiple energy
carriers based on energy hub concept [4]. By introduction of
the matrix model of energy hub and a dispatch factor for deter-
mining the contribution of each converter from different inputs,
they submitted an optimization framework for optimal power
dispatch problem. The same authors [5], have offered optimal
power flow problem between different energy infrastructures,
such as electricity, natural gas and district heating networks
in the context of interconnected energy hubs for multi-carrier
energy systems. Also, the same authors [6] have presented a
structural optimization method to find optimal energy hub cou-
pling matrix based on a specific demand and objective functions.
Such models were offered in steady-state condition and time-
dependent parameters were not considered. Then Geidl [7] has
wrapped up the above content. In this research general frame-
work for energy hub modeling has provided for multi-carrier
energy system management. Also, multiple energy carrier dis-
patches and optimal power flow in both operational and struc-
tural optimization as well as adding a storage system and taking
the time dependency into account have been discussed along
with some practical examples.

In general, planning and management of energy hub in the
form of an optimal scheduling problem can be divided into two
general categories based on the time horizon of the study:

- Short term: operational optimization

- Long term: optimal planning

In the first case, the goal is usually the optimization of the
performance and creating an operational program for the var-
ious components of energy hub and has a short time horizon
from one day to one year. In this case, the smaller time step is
not considered and usually, an hour or a quarter of an hour for
time steps can be considered to be able to ignore the dynamics
and transition of various components of the energy hub. In the
second case, the target is usually the optimum structure design
of the system; for example finding the optimal size of equipment
for a specific demand and considered objective functions. This
case has a longer time horizon from one year up to 20-30 years
which corresponds to a useful lifetime of the system. In this case,
in addition to system operating costs, initial investment costs
are also taken into account and the system is evaluated during
its lifetime.

Such structures need a comprehensive model and an opti-
mization framework. In energy hub models, the objective is
usually minimization of functions such as cost, emissions, pri-
mary energy consumption, peak demand or maximizing func-
tions such as profit, the share of RES and customer satisfactions
subjected to various operational and structural constraints. A
general framework for energy hub optimization models can be
considered as the following relationships:

\[
\text{minimize } f(I_h, F_a, v_{hak})
\]

Subject to:

\[\begin{align*}
I_h - C_h I_h &= 0 \quad \forall h \in H \\
G_a(I_h) &= 0 \quad \forall h \in H, \forall a \in A \\
I_{h,\text{min}} &\leq I_h \leq I_{h,\text{max}} \quad \forall h \in H \\
I_{h,\text{min}} &\leq v_{hak} \leq I_{h,\text{max}} \quad \forall h \in H \\
0 &\leq v_{hak} \leq 1 \quad \forall h \in H \\
&\forall a \in A, \forall k \in k_a
\end{align*}\]

(1)

Where \( H \) is the number of energy hubs in a network of inter-
connected hubs. \( A \) is the number of energy carriers. \( K_a \) is
the number of converters that can use energy carrier as input. With
this description can be said that \( I_h \) and \( I_h \) are input and output
matrixes related to the energy hub of \( h \), coupling matrix \((C_h)\)
is used to link different energy carriers at the input and output.
\( F_a \) is the power flow of energy carrier. \( G \) is a set of networks
equations and constraints. \( v_{hak} \) is dispatch factor that defines
the dispatch of an energy carrier to the converters and their
sum is equal to 1 for energy carrier. So \( \sum_{k \in K_a} v_{hak} = 1 \). Therefore
relations (1) to (6) is a simplified representation of the general
framework for optimizing energy hubs by varying \( I_h \), \( F_a \) and
\( v_{hak} \). Obviously, other relationships such as storage equations,
reliability indices, etc. can be added to these relationships, but
have been neglected here for simplicity.

In general, these relations are in the form of an optimization
problem with the objective function, equality and inequality
constraints and decision variables. In most cases, due to char-
acteristics of the system, such as nonlinear dependency of the
coupling matrix coefficients to inputs, energy hub optimization
problem is a nonlinear problem. In the case of a convex objective
function and linearized constraints, the problem turns into a
convex problem that can be easily solved using numerical meth-
ods. A common example of this case is the considering of a
linear function for energy costs and assuming constant efficiency
for converters of energy hub. But if the objective function is
concave (or convex in maximization mode), and/or nonlinear
constraints, the problem turns into a non-convex problem and
the solution space is no longer convex. Numerical methods
can be used in this case as well, but it cannot be ensured that
obtained answer is the global optimum solution. So this case
requires more advanced optimization methods.

An optimization problem usually refers to a process for find-
ing an optimal solution among a set of possible answers. An
optimization problem includes a set of objective functions, con-
straints of equality and inequality, and decision variables. Con-
vex optimization refers to finding the minimum of a convex
function (or a maximum of a concave function) among a convex
set of constraints. The main advantage of this type of optima-
ization problems is that the local optimization is the global op-
timal point. In this method, any optimization algorithm that finds
the local optimal point, in fact, has found the global optimal point.
Due to the diversity of issues related to optimization in various
fields of research, several techniques have been proposed for
optimization which in this paper it is not possible to deal with
all of them. In this section only dominate algorithms and meth-
ods have been discussed that are useful for solving problems.
related to MES and therefore energy hub. These optimization techniques are discussed as follows:

- **Linear programming (LP):** LP is a method for finding the minimum or maximum of a linear function on a convex polygon. In the case that objective function and constraints are linear and decision variables are continuous, optimization problem will be a linear problem. Therefore, when the objective function is convex, optimization problem has a unique solution. LP can be considered as the easiest and fastest optimization method and even when there is a large number of variables and constraints the optimization problem can be solved with an acceptable speed. However, in real energy hub model with binary variables and nonlinear relationships, this kind of formulation is less useful [8].

- **Nonlinear programming (NLP):** If the objective function and/or constraints over a set of unknown real variables are nonlinear, optimization problem becomes a nonlinear problem. As mentioned, due to the structural characteristics of MES most optimization problems related to these systems are nonlinear in nature. Such nonlinear structures have been formulated by Geidl and Andersson [4–6] for optimal dispatch, optimal power flow, and structural optimization of energy hub, respectively. Other examples can be seen in Table 1. The existence of nonlinear terms in the model leads to a reduction of assurance in achieving a global optimal solution and such a problem may be convex or non-convex. Even if be convex, it is still difficult to solve. Some common methods for solving nonlinear programming are general reduced gradient method and sequential quadratic programming [8].

- **Integer programming (IP):** All or some of the variables are integers. A widely used form of IP is a mixed integer linear programming (MILP). This is similar to linear programming with the difference that in addition to continuous variables, the problem has integer and binary variables. Similar to linear optimization, MILP aimed at finding the minimum or maximum of a linear function over a space with linear constraints, but the existence of discrete variables lead to discrete and non-convex solution space. Therefore solving this kind of optimization problems is more complex than LP requires special methods such as Branch and Bound [8]. In the energy hub by adding the binary control variables, caused by storage systems or describing the state of equipment using discrete steps such as on/off and standby the LP or NLP problems turn to an MILP or mixed integer nonlinear programming (MINLP). It can be said that most of the issues related to energy hubs are NLP or MINLP. To solve such problems in the energy hub so far two groups of methods have been used. Linear equivalents or linear relaxations to convert the NLP and MINLP problems to LP and MILP problems, respectively. The first case involves the assumption of linear relations rather than nonlinear relationships for restrictions; as an example using constant values for the efficiency of converters. A large number of examples from this application can be seen in Table 1. The latter case includes processes for linear approximation of nonlinear relations using different methods such as iterative Branch and Bound, piecewise constant approximation, Taylor expansion, etc. The examples of this case can be seen in [8, 9]. However, such percussive approximations despite the simplification of computing, reduce the accuracy of the model and does not provide a real vision of the system operational conditions. On the other hand solving MILP or MINLP problem in most real systems is difficult and mainly is nondeterministic Polynomial-time hard (NP-hard) [10], that has not polynomial time algorithm. Such problems may need exponential computing time to solve and solving it may greatly increase the computing time. In recent years new methods for solving such optimization problems have been proposed, such as heuristic methods and artificial intelligence-based techniques.

- **Heuristic approaches:** These methods have been designed to solve the problems that classic approaches are not able to solve them or solving them takes a lot of time. These methods try to solve the problem in a reasonable time and achieving an acceptable solution. This solution is not necessarily the optimal solution for the problem and may be an approximation of the exact solution. Therefore, on some issues, it is difficult to ensure that obtained solution is accurate enough or how much it is important to get an accurate answer. On the other hand, heuristic approaches require a good knowledge of the system. If these methods are designed properly, they are appropriate to reduce the computational load of optimization algorithms and achieving an acceptable solution. The main problems of heuristic algorithms are sticking in a local optimum, premature convergence on these points and dependence on good knowledge of the system. Metaheuristic algorithms have been proposed to solve these problems of heuristic algorithms.

- **Metaheuristic approaches:** These methods are a generalization of the heuristic approaches which can be applied to a wide range of the problems. These algorithms also are approximation optimization approaches that are capable of getting out of the local optimum. In spite of heuristics, these methods require fewer assumptions and information about the system. Metaheuristic usually divided into two general categories; Trajectory-based or population-based. Trajectory-based metaheuristics also known as single-solution algorithms which are based on a single solution during the search process and focus on improving a solution and usually provide a single solution. Common trajectory-based metaheuristic algorithms, are Tabu search and Simulated Annealing algorithms. Population-based metaheuristics use multiple solutions and they often use population characteristics to guide the search. These algorithms create the initial set of potential solutions and are gradually improving these initial solutions and finally provide a suitable solution for the optimization problem. The most known population-based metaheuristics are Genetic Algorithm and, Ant Colony Optimization, Artificial Bee Colony Algorithm, and Particle Swarm Optimization. Metaheuristic algorithms can be combined with other optimization approaches, which lead to the Hybrid Metaheuristic. Also, multiple metaheuristics can be used in parallel which leads to Parallel Metaheuristic.

- **Parallel Computing:** A process in which a large problem can be divided into several smaller sub-problems which can then be solved at the same time. Reduction in the computation time, the possibility of resolving the larger issues, and overcoming the limitations of memory are the advantages
of this method. Two known examples of these methods are Dynamic Programming and Monte Carlo Simulation.

- Multi-objective optimization: Most optimization methods mentioned above are used to solve single objective problems, while most problems in the real energy hub models are multi-objective problems and usually these objectives are in conflict. Minimizing energy costs while maximizing comfort within the energy hub and taking into account all these possible connections leads to an increase in model complexity and optimal decisions need to be taken in trade-offs between two or more conflicting objectives. There are different methods for solving multi-objective problems include Bonded objective, Absolute priorities, Goal Programming, Goal attainment, Weighted sum method, Pareto-based optimization that last three of these are most widely implemented. The main idea behind goal attainment method is to find solutions that satisfy a predetermined objective. A practical example of this approach in the energy hub in order to solve a multi-objective power flow optimization problem in a network of interconnected hubs can be found in [11]. Weighting method is to assign a weight to each of the goals which lead to the creation of an objective function of the sum of previous functions. This method can be considered as the simplest method of solving multi-objective functions. However, this method is applicable only in the convex problems. Also, the difficulty of adjusting the weights and not provided optimal trade-off between conflicting objectives are the disadvantages of this approach. The shortcomings of these methods can be compensated with Pareto-based optimization. The Pareto method is a common method for solving multi-objective problems and identifying suitable trade-off between different objective functions. One solution is Pareto optimal, if improving an objective function, is not possible without degrading the other objective functions. Without any external criteria, all Pareto optimal solutions can be equally acceptable. Therefore, solving the optimization problem requires finding all Pareto optimal solutions. All these optimal solutions are not easy to achieve and so in recent years meta-heuristic methods application for solving multi-objective problems has been increased and a large number of multi-objective metaheuristics have been presented. Some of these methods are non-dominated sorting genetic algorithm (NSGA/NSGA-II), multiobjective simulated annealing (MOSA), and multi-objective Tabu search (MOTS).

As mentioned, these methods and structures have been commonly and widely used in energy hub models, but other specific methods are also used in energy hubs. For example, Arnold et al have used a model predictive control (MPC) method for centralized control [12] and distributed control [13], for a network of interconnected hubs by considering the dynamic of energy storage systems as well as energy prices and demand forecast. Bahrami et al [14] have modeled the interactions between electricity and gas companies with a set of the interconnected hubs as an ordinal potential game in a smart environment. Sheikhi et al [15] have used reinforcement learning (RL) algorithm for optimal management of a residential smart energy hub to reduce energy costs and taking into account the effect of the customers loads shifting factor on their satisfaction. The summary of models and optimization techniques used in energy hub models can be seen in Table 1.

Almost all the models presented in Table 1 and discussed so far are deterministic models. However, another important part of the optimization approaches is non-deterministic optimization such as stochastic optimization. Another side of realization coin is the complexity of modeling. This means that taking into account the real situation of the system in many cases leads to the complexity of the problem and requires advanced modeling and solving approaches. Lack of attention to actual conditions leads to unrealistic modeling, which results in illusory and inaccurate results. In energy hub models, lower accuracy occurs in two modes. The first model is modeling simplifications and the second one is ignoring the uncertainties. For example, the efficiency of a CHP is a nonlinear function of its operating point and taking into account this function leads to the complexity of modeling and problem solving is more difficult. In the energy hub models, this efficiency has been usually considered as a constant value. This simplicity leads to a reduction in the accuracy of the model.

In the literature so far, some limited research have been provided to improve the realistic modeling of the energy hub. A new formulation for energy hub has provided in [8] to improve some operational constraints such as minimum operating time after equipment start-up, storage losses and piecewise linear approximation of non-linear efficiency curve. Two limitations of energy hub models, namely the problem of distinguishing the appropriate port for connecting different components (especially RES) and the problem of impossibility of evaluating the bi-directional power flow have been addressed in [16]. In this paper, the authors have used a modified version of the energy hub model by using the graph and network theory to address the above limitations in steady state. The authors in [17] have provided a general heuristic optimization framework to solve optimal power flow problem in the network of the interconnected energy hubs which can be used with all evolutionary methods and provides the possibility of taking a variable function for efficiency into account. The existence of various connections within the energy hub and taking into account all these possible connections leads to an increase in model complexity and difficulty in optimal control of the system. Authors in [18] have provided a method for modeling such a complex energy hub taking into account all the possible connections between system components which lead to increased reliability of demand supply.

As mentioned, the latter case that leads to a reduction in the accuracy of the model is ignoring the impact of uncertainties of the input parameters in the optimization process. For example, demand for the future periods is uncertain and taking into account the known demand or a complete forecast of the demand will cause illusory results and will not be considered risks of the changes in demand levels. Other examples include the energy price, weather conditions, RES output, consumer behavior and so on. Energy hub models should consider these uncertainties for achieving a comprehensive and realistic model of the sustainable energy systems in the future. Different approaches for addressing uncertainties is discussed in the next section.
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<td>MILP</td>
<td>GAMS optimization solvers</td>
<td>Optimal operation of a network of EHs</td>
<td>2015</td>
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<td>[9]</td>
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<td>One day</td>
<td>MILP</td>
<td>GAMS optimization solvers</td>
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<td>2015</td>
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<td>[8]</td>
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<td>One day</td>
<td>MILP</td>
<td>GAMS optimization solvers</td>
<td>Optimal operation of a network of EHs</td>
<td>2015</td>
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<td>[7]</td>
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<td>One day</td>
<td>MILP</td>
<td>GAMS optimization solvers</td>
<td>Optimal operation of a network of EHs</td>
<td>2015</td>
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</table>

Table 1. A summary of models and optimization techniques used for energy hubs in the literature
3. UNCERTAINTY CONSIDERATION IN MODELING

In general, it can be concluded from the above discussion that an optimization problem is essential to obtain optimal planning and management of the energy hub. Under certain conditions, this optimization problem has an optimal and unique solution. According to the problem condition, this solution can be obtained from different optimization methods. However, for nonlinear, multivariate, and multi-objective functions finding an optimal solution is not an easy task. In these cases, more advanced methods can be used to solve the optimization problem and finding an approximation of the possible optimum solutions. All of these occur in deterministic conditions but when the input variables of the optimization problem are not constant small changes in any of them can change the optimization results.

In an actual environment, usually many variables such as demand, the price of energy, solar radiation, etc., are not constant and have random and variable behaviors. This random behavior and fluctuating nature, lead to uncertainty in the modeling and ignoring the impact of these uncertainties leads to inaccurate models and illusory results. Therefore, the impact of these uncertainties should be considered in the optimization. In general, uncertainties have different resources and hence different types. Each of these types of uncertainties may affect one or more steps of the modeling and optimization processes. Identifying the type of uncertainty helps to better understand their effects on system modeling and performance. The authors in [44] provide a category for a variety of uncertainties that is summarized in Fig. 3.

Variability uncertainty caused by the inherent variations in nature, behavior changes, changes in technology and organization. Knowledge uncertainty is due to limited knowledge about scheduling and making decisions about the system. This type of uncertainty can arise due to the perception of decision makers and planners about scheduling content include system realities, uncertainty, and boundary conditions. On the other hand, knowledge uncertainty can be caused because of the model features such as:

- Lack of knowledge of input parameters
- The uncertainty inherent in the model input data
- Measurement errors
- Insufficient understanding of the processes and system conditions such as physical and dynamic processes
- Simplification and approximations
- Hardware and software errors
- The accumulation of all these

Linguistic uncertainty is caused by the information expressed by the human that is in linguistic terms and inherently uncertain. Language restrictions in precisely quantifying a parameter (Vagueness), a word that has several meanings and its exact meaning may not clear (Ambiguity), and loss of detail in specification or generalization of some concepts (Under specificity) are some of the linguistic uncertainty items. The decision uncertainty is due to the ambiguity in the definition, quantification and comparing objectives and performance indicators. Procedural uncertainty arises from the imbalance between available resources and time.

Accurate recognition of the uncertainties and their impacts increases the accuracy and realizing of the model. However, this uncertainty modeling in a multi-carrier, multi-objective, multi-period and multi-criteria framework is not an easy task. Different methods for addressing and handling of the uncertainties have been presented in the literature so far. The authors in [45, 46] have provided a category of uncertainties modeling techniques in the process of decision-making in power and energy systems that is summarized and schematically shown in Fig. 4. The main similarity between them is that all of these methods attempt to quantify the effects of uncertainty on the output parameters and the main difference is in the expression methods of uncertain parameters.

Probabilistic methods use probability density function (PDF) to express uncertainty and are used when sufficient data are available and PDF of uncertain parameters is known. Fuzzy methods use the membership function (MF) to display uncertainty. Unlike previous methods, Information gap decision theory method does not use the PDF or MF and in fact, reveals the error between the parameters values and their predicted values. Robust optimization method uses uncertainty sets to describe the uncertainty and this model actually is used for the worst case. Finally, interval method uses interval values with lower and upper bounds to display uncertainties. In the following sections, each of the mentioned methods and their application in energy hub models is described.

A. Probabilistic approach

Deterministic modeling does not provide a real image of the dynamic behavior of the real world. Dantzig [47] and Beale [48] (1955) provided the stochastic programming model independently that in numerous research have been studied and expanded [49].

Probabilistic methods use statistical distributions such as Weibull distribution for wind speed and the normal distribution for load changes to express uncertainty. A heuristic optimization method known as multi-agent genetic algorithm was used in [50] to solve an economic dispatch problem in a multi-carrier system with integrated energy hubs. In this study, uncertainty related to wind power generation was modeled by Weibull distribution as probability distribution function and performance of proposed method was compared with methods such as GA and PSO. A DR program based on minimizing the cost of a multi-energy system by considering the level of customer satisfaction has
been provided in [51]. Uncertainties arising from the behavior of consumers in multi-energy system management is considered. These uncertainties include the percentage of consumers who participate in DR programs, the percentage of consumers who have the ability to switch between different energy carriers and consumers who do not have the ability to switch between different energy carriers and may control their demand. A normal distribution has been used to display the uncertainties and producing related scenarios. An optimization model is presented in [52] for a hybrid energy system by considering the various uncertainties based on statistical data. The authors have used binomial distribution for solar and wind power generation uncertainties, a normal distribution for demand and Bernoulli distribution for inaccessibility of production units. The impact of RES on system planning has been evaluated by proposed model in the presence of uncertainty.

Probabilistic methods are usually divided into two main groups: numerical methods and analytical methods. Numerical methods are mainly based on random sampling and Monte Carlo Simulation (MCS) is the most important one. This method is used when the system size is large or have many uncertain parameters or have a large number of non-linearity. A model for optimal management of energy hub has been provided in [53] under uncertainty and changes in electricity prices. Prices of all input and output energy carriers are regarded as a random variable and MCS considers thousands of possibilities in prices of these carriers. An optimal sizing of a system of interconnected hubs taking into account electric and gas networks physical constraints, reliability indicators and environmental issues has been provided in [54]. The impact of uncertainty in electrical and thermal loads, as well as uncertainty in energy prices, has been studied using Monte Carlo simulations and the performance of interconnected energy hubs system has been proven even in the presence of these uncertainties. A multi-criteria study for ranking different RES at the national level in Scotland has been provided in [55] and the impact of input data uncertainty on the results has been investigated. For this study different technical, economic and environmental criteria have been used that each of these criteria has a wide variation range. To consider this type of uncertainty, the amount of each of these criteria is shown by a statistical distribution and the MCS is used to perform the multi-criteria evaluation. The study showed that the result of ranking extremely depend on uncertain parameters variations. The impact of energy hub on reducing energy costs and increasing profits in the presence of uncertainty the electricity price has been studied in [56] by using MCS and creating five different scenarios for different levels of uncertainty. In general, it can be said that the MCS is easy to run, flexible, usable in convex and non-convex issues and supports from all types of PDFs. Instead, the number of required simulations increase by increasing the solution space’s degree of freedom. So thousands of simulations get the exact answer is needed and thus the computational load of this method is high.

Analytical methods mainly are based on approximation and use mathematical expressions to display the system’s input and the output. The first of these methods are based on linearization and their goal is to make output PDF from input PDF. Methods such as Convolution, Cumulants and Moments, Taylor series expansion and First Order Second moment are in this category. In these methods, mathematical operations such as convolution, calculating the coefficients of expansions and so on are used to get output parameter’s PDF from the input parameter’s PDF.

All the above methods are common in linearization and this linearization is reliable when the expansion of error is approximated using an appropriate linear function. These difficulties and inaccuracies in linearization process lead to the development of other types of methods. These methods are based on creating a good approximation of input parameter’s PDF using the appropriate samples. Point estimate, Unscented Transformation and Scenario-based decision-making methods are in this category. Point estimation method includes the use of sample data to calculate a value as the best estimates and forecasts of a parameter. This method is non-repetitive, simple to use and does not have the problem of convergence. However, this method doesn’t provide information about the exact shape of output’s PDF and only gives the mean and standard deviation. This method, like other probabilistic methods, needs PDF of uncertain parameters and gives a better answer for non-skewed PDFs. Unscented Transformation method is appropriate to consider the correlation between random variables and is a reliable method to calculate the output of random variables under a set of non-linear transformations. This method is highly functional and its accuracy does not reduce by increasing the number of random variables but its run time depends on the number of random variables and also needs to PDF of input parameters.

Uncertain parameters have numerous realizations which are impossible to consider all of them. However, they can be converted into a number of limited and countable scenarios. This process is the basis for the scenario method. This method converts continuous space to the limited number of discrete scenarios with the related probabilities and so this method enhances computational efficacy. However, the scenario-based approach is an approximate method and requires the statistical informa-
A model for mid-term management for an energy hub taking into account uncertainty of wind generation and electricity market with the objective of minimizing costs has been provided in [59]. In this study, Auto Regressive and Integrated Moving Average (ARIMA) and Auto Regressive and Moving Average (ARMA) time series have been used to generate different scenarios. ARIMA method for prediction of the price of electricity and ARMA method for prediction of the wind speed used and the generated scenarios used to model the problem in the framework of a stochastic programming model. By using the mean and variance of input data, ARIMA method can be used to predict future changes in parameters. Usually, the models that use ARIMA method need to be combined with a scenario reduction method. Here, wind speed and price scenarios for electricity production are completely independent of each other and should be considered all the possible combinations of them. Therefore, because of the increased number of scenarios, computing time increases and authors has used a scenario reduction method. The impact of DER, ESS, and DR on the performance of a commercial energy hub participates in DR programs in addition to shifting or reducing their demand have the ability to switch between different energy carriers. In this paper, some of the demands have been considered as dependent loads which can use different energy carriers to deliver a particular service. Therefore, the management of such loads by consumers to choose between different carriers will lead to uncertainty. The results show that the uncertainty in the random behavior of consumers in the DR program is effective on the share of energy carriers in the demand supply. Increasing the share of loads that are controlled by consumers increases the impact of random behavior of consumers in system performance and leads to increased operating costs of the system.

### B. Fuzzy approach

As mentioned, because of increased uncertainties in the energy systems, dealing with these uncertainties has become one of the main issues in MES modeling and scheduling. The fuzzy sets approach is an efficient method for dealing with these uncertainties and modeling the real behaviors. On the other hand, energy hub directly interacts with natural processes and human behaviors and so modeling this behavior with certain and determined values is not effective because usually human behavior and natural processes are not predictability deterministic and controllable. Whereas that one of the main applications of fuzzy logic is quantifying of the qualitative and linguistic terms so using this method for expressing the uncertainties helps to achieve the realistic models and extraction more accurate results. Fuzzy sets use the concept of fuzzy membership functions such as triangular, trapezoidal membership functions and Gaussian fuzzy set for uncertainty modeling. Fuzzy logic-based models are used mainly in three ways: fuzzy models, hybrid models and multi-criteria decision models that are shown in Table 2. Fuzzy models are usually used in order to realize system conditions and characterization linguistic terms. These models have applications such as expressing expert linguistic terms, data capturing, finding the relative importance of variables, resources grouping, data clustering, and predictive purposes. Hybrid models have a variety of applications in energy systems, from designing of control systems to locating and determining of the optimal combination of RES. Multi-criteria models usually have a lot of computing load and have diverse applications such as performance optimization, system control and emissions and financial risks reduction [62].

In this regard, a combination of interval linear programming (ILP) model and fuzzy logic have been presented in [63] to express multiple uncertainties for using in long-term planning and capacity expansion. ILP is an approach for considering uncertainties as interval within the constraints or objective function.

### Table 2. Fuzzy logic based models in energy systems

<table>
<thead>
<tr>
<th>Fuzzy Approach</th>
<th>Fuzzy Models</th>
<th>Fuzzy Decision Models</th>
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<tr>
<td>Fuzzy regression</td>
<td>Fuzzy ANP</td>
<td>Fuzzy-VIKOR</td>
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<tr>
<td>Fuzzy AHP</td>
<td>Fuzzy TOPSIS</td>
<td>Fuzzy TOPSIS</td>
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<tr>
<td>Fuzzy asymptotic design (FAD)</td>
<td>Fuzzy particle swarm optimization</td>
<td>Fuzzy support vector machine</td>
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<tr>
<td>Fuzzy gray prediction</td>
<td>Fuzzy bee optimization</td>
<td>Fuzzy particle swarm optimization</td>
</tr>
<tr>
<td>Fuzzy clustering</td>
<td>Fuzzy cuckoo search optimization</td>
<td>Fuzzy ant colony optimization</td>
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<tr>
<td>Fuzzy expert system (FES)</td>
<td>Fuzzy quantum particle swarm optimization</td>
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</table>
Therefore it has less complexity and computational load but because of the lack of time dependencies consideration, its conformity with actual results and data is less than other methods.

Stochastic methods use probability distributions. The two-stage stochastic method uses a two-step decision-making process that in the first stage a decision is taken based on future events with uncertainty and when it happened, a corrective action is done. But this method requires a clear PDF of the input parameters and sometimes PDF of these parameters are not available or are not accurate or it is not possible to use them in large-scale problems. To solve this problem authors in [64] have used a combination of ILP, two-stage stochastic programming (TSP) and fuzzy logic for optimal planning of a regional energy system that enhances the functionality of the model to deal with uncertainties. Also, TSP has some problems for reflecting the dynamics of the energy system. Multi-stage stochastic programming (MSP) is an extension of the TSP that is able to display the dynamic characteristics. Uncertainties in the MSP are modeled through multi-layer scenario trees that lead to flexibility in decision-making processes and scenarios modeling, especially for the large-scale problems. A combination of interval fuzzy linear programming (IFLP) and MSP has been provided in [65] that uncertainties have been considered by using fuzzy sets, probability distributions and interval values in the framework of an MILP problem. The proposed model has been applied to a case study based on energy and environmental management. The results help to decision makers in the resources allocation, pollution reduction planning and resulting in a trade-off between system cost and environmental requirements under multiple uncertainties. So a combination of fuzzy logic models and stochastic programming have been used for considering the multiple uncertainties in optimal scheduling of energy systems.

Energy storage systems are used to benefit from the excess energy produced by RES at any time. Optimal operation of this complex need for a control strategy to coordinate their performance. Battery charge and discharge control program has been modeled in [66] by using fuzzy logic. In this study, the system has been considered as a set of elements that each element has been assumed as an agent that can control their behavior according to environmental conditions and changes. An optimal operation model for a hybrid system based on fuzzy logic has been proposed in [67] which is the combination of PSO and fuzzy theory. In order to control fluctuations and changes in the system, a fractional order fuzzy PID controller has been used to control charging/discharging of battery and production of a diesel generator. The proposed algorithm has high robustness against changes in input parameters and increases system reliability. A fuzzy expert system has been used in [68] to control the output of storage system in the context of a GA-based optimization algorithm and is an example of combining fuzzy theory and GA. This model leads to lower system costs compared using the charging/discharging the battery control based on a threshold.

With the growing penetration of RES in the energy system, output fluctuations of these sources can cause problems such as variations in system voltage and frequency. A method for leveling the renewable energy output changes by using fuzzy control has been presented in [69]. It also controls the power output of PV according to system conditions and taking into account the solar radiation in order to make maximum use of sunlight. As a result, presented fuzzy control make a trade-off between reducing fluctuations and maximizing the use of radiation. A PID controller based on fuzzy logic has been used in [70] to minimize the frequency and voltage deviations in a hybrid system. PID controller alone cannot prove well act under conditions of intermittent wind speed and even adding storage system also doesn’t make the system robust against sudden changes in wind speed. But adding a fuzzy controller to the system leads to the successful performance of the system under load and wind speed uncertainty conditions.

Using interval methods along with fuzzy methods have become very popular that describe uncertainty variations in the form of intervals with the upper and lower boundary values. In the literature, several interval methods such as fixed intervals or functional intervals are used to display uncertainty. An interval fuzzy approach has been proposed in [71] to change the boundaries in accordance with the system dynamic and confidence level. In this paper, a scenario-based energy management system for a microgrid has been provided that scenarios are produced using interval fuzzy model and taking into account the uncertainty arising from demand and RES. Fuzzy linear programming (FLP) is an effective method for quantification of ambiguity and uncertainty of information on energy management system that has been developed based on the combination of interval and fuzzy sets but this method for large-scale systems become more complex. Interval linear programming (ILP) makes it easier to address the uncertainties but when the uncertainty level of parameters is high does not provide a reasonable solution. So the combination of these two models is used to benefit from the advantages of both of them. A combination of ILP and FLP has been used in [72] to show the uncertainty in an energy system management problem with the objective of minimizing energy and environmental costs. A hybrid model of ILP and FLP has been presented in [73] that by using Type 2 fuzzy sets increases the amount of fuzziness that can be faced with a higher level of uncertainty and improve the performance of IFLP hybrid model. This method offers a realistic simulation of the energy flow in the system that can be used in decisions related to the capacity expansion and resource allocation. A combination of fuzzy programming models and stochastic linear programming has been presented in [74] that express uncertainties by using probabilistic distribution and Type 2 fuzzy sets. This method without unrealistic simplifications improves the system efficiency and energy security.

In short, it can be said that fuzzy logic has wide applications in modeling and scheduling of energy systems. One of the main applications of fuzzy methods is the modeling and expressing of the uncertainties and ambiguities of energy systems. The use of fuzzy logic to deal with these uncertainties leads to model realization and obtaining more accurate results and also facilitates optimization and decision-making processes. Fuzzy logic controllers are widely used in controlling the power flow of energy systems. As well as hybrid models, including the combination of fuzzy methods, heuristic techniques or multi-criteria decision-making approaches have been used in literature and their number is increasing. The combination of fuzzy models with heuristic techniques despite the complexity of such methods leads to simplicity in using (user-friendly) and the accuracy of these methods. The combination of fuzzy models and methods such as time series, neural networks, and regression are used in prediction models to increase the accuracy of the model. Despite the high potential for application of fuzzy sets in energy hub, however, so far there has not been any research on the use of fuzzy methods in optimal scheduling of energy hub. Therefore, there is a great potential for the use of fuzzy models in the management of energy hub.
C. Information gap decision theory

When there are not sufficient data about the uncertain parameters, PDF or MF cannot be used and methods like Information gap decision theory (IGDT) can be useful. The purpose of this procedure is to reveal the results of the difference between actual and predicted values of uncertainty. In decision-making based on IGDT, two cases occur; risk averse or risk seeker [75]. In the first case (risk averse) the decision-maker is looking for a robust decision against possible errors of uncertain parameters prediction. This decision is created when the objective function is protected against the maximum variations radius of uncertainty. In other words, a forecast of the uncertain parameter is performed and for an authorized range for the objective function, the maximum permissible variation range of uncertain parameters is calculated. Risk seeker case tries to find the minimum variations range of uncertain parameters. This means that for a degree of freedom for the objective function, it find the minimum variation range of uncertain parameters. In other words, decision maker decides in the worst predicted case. Soroudi and Keane [76] have provided a good example of the use of IGDT in optimal management of energy hub. In this study, by taking into account the wind power generation, electrical and thermal loads as uncertain parameters, a prediction of these parameters has been carried out and the maximum allowed variations intervals for this parameter have been calculated in different scenarios. Finally, the necessary measures to achieve a risk-averse management of energy hub have been provided. In [77], risk constrained scheduling of head and power producer is formulated using IGDT considering risk-averse and risk-seeker options.

Other examples of the use of IGDT in energy systems [78], energy markets [79, 80], GenCos scheduling [81, 82], power distribution network dispatching [83, 84], UC [85], self-scheduling [86] and so on can be found in the literature and take into account the proven effectiveness of this method there is a good potential for application of this approach in the management of energy hub.

D. Robust optimization

Robust Optimization (RO) are used when the statistical information of input parameters is insufficient and there is no possibility of extracting PDF of uncertain parameters. In this method instead of PDF, the interval values are used for displaying uncertainty and the problem is solved for the worst case at any interval. Thus, this method is very conservative. In the case of a parameter that is characterized by uncertainty, robust optimization ensures the decision maker that even if there are errors in the prediction of uncertain parameters, the objective function value will remain optimized. A robust optimization has been used in [87] to consider the uncertainty arising from demand and cost (such as emissions tariffs) and the price of electricity and gas in hybrid energy system planning problem for a commercial building. The authors in [88, 89] have developed a model for optimal management of a micro-grid based on robust optimization. In this paper, the uncertainty arising from wind power generation and load are displayed as prediction intervals and up and down boundary values have been predicted by a Non-dominated Sorting Genetic Algorithm (NSGA-II) - trained Neural Network (NN). Results of RO-based optimal management have been compared with the results of optimal management based on expected values in terms of performance and reliability of the system in conditions of happening different uncertain events. Results indicate better performance for RO-based optimization. In order to secure the operational model of an energy hub in the face of uncertainties caused by the deviation of the equipment efficiencies from their nominal values, a robust optimization method has been used in [90]. Authors have shown that for securing the model against uncertainties operating costs of the system will rise, but instead, the possibility of complete demand supply grows. As a result, a robust optimization is a conservative approach which increases the robustness of the system against the uncertain parameters, but the result is not always the optimal solution.

E. Interval analysis

Interval method usually is used when we have variations on the interval of uncertain input parameters. In this method, the upper and lower boundary values for output parameters can be obtained by defining upper and lower boundary values for uncertain input parameters. Interval numbers have been used in [91] for considering the uncertainty of the wind power in an operational optimization problem of an interconnected electricity-gas energy system taking into account DR programs. In this study, the power flow problem in the interconnected system has been solved with the objective of minimizing the operating costs. As previously mentioned, ILP is a way to handle uncertainties that are displayed as intervals and without a known statistical distribution or membership functions. In this way, uncertain parameters are displayed by crisp intervals with fixed lower and upper bounds. In the real world, uncertain parameters are changing and these changes are affected by the various parameters and so presenting this changes with fixed intervals does not show the impact of these changes on system performance. An effective way for considering such changes is the functional intervals. In this way, lower and upper bounds are shown as a function of effective parameters and therefore the interval boundaries of uncertainty parameters are variable. An application of the functional intervals method in urban energy system planning, with different primary energy sources, has been provided in [92] to determine appropriate energy resource alternatives. In this paper, the boundary of the electricity price and purchased power have been considered as a function of energy prices. In this model, the intervals have been assumed to be a linear function of a parameter but in many real cases these functions may be non-linear relationships or intervals may be a function of several effective parameters. Therefore, this model can be used for taking into account such uncertainties in energy hubs management. Interval methods, usually are used in the form of hybrid models with other methods for the consideration of uncertainty. In this case, multiple uncertainties can be considered as statistical distributions, membership function, interval values or a combination of them in energy hubs modeling. Examples of the use of this hybrid modeling method can be found in [71, 73, 74].

Interval optimization is easier to use in engineering systems. Its computational load is less than the stochastic methods and instead of finding the worst case (RO), recognizes the optimal interval of the objective function. But in this method, there is no possibility of taking into account the correlation between uncertain parameters.
F. Z-numbers

The concept of Z-numbers that is related to the reliability of information was introduced by Zadeh in 2011 [93]. As discussed in the previous sections, different methods are used for the uncertainty modeling. However, the Z-number offers a better representation of the uncertainty of the real world. A number is displayed as a binary pair $Z = (A, B)$ that first component, $A$, is a restriction on the value of an uncertain parameter, $X$, and the second one, $B$, shows the reliability of $A$. In fact, $A$ is a restriction on the amount that $X$ can take and $B$ is a prediction of reliability of $A$ and is considered as probability measure of $A$. This concept represents reliability of input data that can be used in many areas such as decision making, forecasting, risk assessment, economics, engineering, and so on. For example, if we speak about oil prices in the near future. We say that the price of oil in the next year is about $40. Expression of this concept by Z-numbers is in the form of $Z = (about \$ 40, most likely)$. So, many linguistic terms can be displayed using Z-numbers theory. The Z-number is trying to consider the input information uncertainty and formulation the remarkable ability of the human mind in decision making in the ambiguous and uncertain environment. Therefore, it can be said that the evolution of the uncertainty models is in the form of real numbers, intervals, fuzzy numbers, random numbers, and now is Z-numbers. In fact, can be said that most concepts and numbers in the real world are in the form of the Z-numbers and all of the methods that were used for the realization of the models have been a simplification of Z-numbers concept [93]. One of the main problems associated with Z-numbers is information processing and solving the problems related to them. So many different works have been presented in the literature to simplify the calculation and application of the concept of Z-numbers. The incentive to use Z-numbers, simple examples of its concept, calculations with simple operations and concepts such as Z-numbers ranking have been presented by Zadeh in [93]. The authors in [94] have provided a method for converting the Z-numbers into fuzzy numbers. In this way, the second component of Z-number has been defuzzified to a real value and by multiplying the numeric value by the first component, a fuzzy number is obtained and considered as the representative of Z-number. By using this method, it becomes easier to use Z-numbers in calculations, but the original information of Z-number is lost. The same authors, based on their approach have developed a framework of multiple criteria decision-making in uncertain environments in [95], where the weight of each criterion has been described by using a linguistic term and considered as Z-numbers. Then Z-numbers have been converted to the crisp numbers and evaluation of the alternatives has been done based on this numbers. The authors in [96] have provided a model of analytic hierarchy process (AHP) based on Z-numbers to deal with problems related to decision-making based on linguistic terms. In this study, the weights of criteria have been described as Z-numbers and the proposed method in [94] has been used for converting Z-numbers and then comparing process has been done in the field of real numbers. A model of decision-making in an uncertain environment based on information described by Z-numbers has been provided in [97] and after converting the Z-numbers to fuzzy numbers, the decision has been carried to choose alternatives using a fuzzy measure and within the fuzzy framework. A method for decision making under interval set-valued fuzzy and Z-numbers uncertainties has been presented in [98]. Yager [99, 100] has offered a new methodology and application of Z-number in different fields by using sample distributions and certain assumptions. Also in [101], the issues of continuous Z-numbers calculation and examples of their use in various fields has been discussed. Applications of Z-numbers in calculating with words (CWW) and the issues and problems related to their integration have been offered in [102, 103]. Different ways for evaluation of a Z-number with a reduction in the computational complexity has been presented in [104]. Authors in [105] have provided a method for direct calculating of the discrete Z-numbers to avoid simplifications and data loss of Z-numbers. A model for the integration of LP methods and Z-numbers in the form of a Z-numbers based LP (ZLP) problem has been presented in [106]. In this paper, the reliability of input data in the LP model has been considered as Z-numbers. The authors in [107] have used the Z-numbers for evaluating the effect of system resilience on risk and hazard parameters in a petrochemical unit.

As can be seen, different studies have been carried out on the Z-number concept and efforts for developing the concept and facilitating the calculation of Z-number is ongoing. Z-number is a comprehensive concept for taking into account the uncertainties of the real world and the reliability of input data that improves the accuracy and the ability of the model for decision making in the uncertain environment. Since the energy hub directly faced with the uncertainties of the input data, the use of Z-numbers for more realistic energy hub models is inevitable and by developing and improving the arithmetic of Z-numbers its applications in energy hub models can contribute significantly to realize these models.

4. GENERAL COMMENTS AND SUGGESTIONS FOR RESEARCHERS

As described, several methods have been presented for modeling uncertainties in the literature and these methods have been used in various fields. An overview of various methods for modeling uncertainty in energy systems, the advantages and disadvantages of them, and their application in different energy systems were provided to prove the performance of the different methods in dealing with different uncertainties. Table 3 summarizes the use of these methods in the energy hub. As can be seen, despite the necessity of uncertainty modeling in optimal management of energy hubs, so far very little studies have been done in this area. It represents a weakness in energy hub models in the literature, because of deterministic modeling of the energy hub management problem. At the same time represents a high potential for research in the field of uncertainty modeling in MES in the content of energy hub.

Therefore some of the main uncertainties in energy systems that most commonly mentioned in models and their modeling approaches using a variety of methods are proposed in Table 4. As can be seen, the scenario-based method has many applications in the modeling of uncertainties and even in dealing with multiple uncertainties. The reason is easy applicability of this method and low computational load (with the exception of cases where the number of scenarios is very large) compared to other models. Fuzzy methods have more application in the wind and solar power generation uncertainties modeling. As well as it is a useful method for use in combination with other
methods in hybrid models to consider multiple uncertainties. However, fuzzy methods so far have not been applied in the energy hub models. As shown by increasing the number of uncertain parameters, the complexity of the problem increases and the need to use hybrid methods in dealing with multiple uncertainties rises. Obviously, all this kind of uncertainties listed in Table 4 can also appear in the energy hub modeling. Table 4 helps to select the appropriate method for modeling various uncertainty and illuminates the route for enthusiast researcher for modeling energy hub under different and multiple uncertainties.

### Table 3. Summaries of uncertainty modeling applications in energy hub

<table>
<thead>
<tr>
<th>Uncertainty modeling methods</th>
<th>References</th>
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<tbody>
<tr>
<td>Probabilistic approaches</td>
<td>Monte Carlo simulation (MCS) [53, 54, 56]</td>
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<tr>
<td>-</td>
<td>Point estimation method [108]</td>
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<td>-</td>
<td>Scenario-based decision making [36, 58–61]</td>
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<tr>
<td>Fuzzy approaches</td>
<td>-</td>
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<tr>
<td>Information gap decision theory</td>
<td>-</td>
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<tr>
<td>Interval analysis</td>
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<td>Z-numbers</td>
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<td>Hybrid approaches</td>
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### Table 4. Summary of most popular uncertainty modeling approaches dealing with most common uncertainty types in multi-energy systems

<table>
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<tr>
<th>Most common uncertainty type</th>
<th>Probabilistic</th>
<th>Fuzzy</th>
<th>RO</th>
<th>Interval</th>
<th>Hybrid</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wind speed</td>
<td>[35, 44, 45, 101]</td>
<td>[21, 23, 34]</td>
<td>[8, 50]</td>
<td>[64]</td>
<td></td>
</tr>
<tr>
<td>Solar irradiation</td>
<td>[2, 35, 101]</td>
<td>[21, 23, 34]</td>
<td>[8, 50]</td>
<td>[64]</td>
<td></td>
</tr>
<tr>
<td>Load</td>
<td>[10]</td>
<td>[9, 44, 110]</td>
<td>-</td>
<td>[51]</td>
<td></td>
</tr>
<tr>
<td>Energy carrier price</td>
<td>[10]</td>
<td>[9, 44, 110]</td>
<td>-</td>
<td>[51]</td>
<td></td>
</tr>
<tr>
<td>Longitudinal terms</td>
<td>-</td>
<td>[10]</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>Multiple uncertainties</td>
<td>[41]</td>
<td>[44, 52, 53, 61]</td>
<td>-</td>
<td>[63–65, 72, 73, 115, 116]</td>
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</tbody>
</table>

5. CONCLUSION

In this paper, the importance of addressing the uncertainties in the optimal scheduling of energy hub have been discussed. It is obvious that energy hub, as a concept for integrated management of MES is influenced by several factors that many of these factors are main sources of uncertainties. For example, the growing interest in RES and the increased share of these resources in energy systems due to the fluctuating and uncertain nature of these resources add uncertainties to energy hub models. On the other hand, energy hub has various consumption areas and behavior of consumers in each of these areas is uncertain and leads to uncertainty in the prediction of actual demand of energy hub. On the other side, energy hub for supplying energy demand and selling their excess energy, interact with energy markets such as electricity and gas markets. Due to the structure of these markets, (particularly in competitive and deregulated markets) pricing of energy and the behavior of other market participants are the main sources of uncertainty and so modeling of energy hub can be affected by uncertainties arising from the interaction with these markets. On the other hand, as the energy hubs do not have a limit on the size and can range from a residential building to even an entire city energy system, and so widely associated with environmental and climate issues. Therefore forecasts taken from the environment is also a major source of uncertainty in the energy hub models. Simplifications and approximations in the modeling of energy hub can also lead to unrealistic models and results. These cases are just a few examples of the energy hub interaction with uncertain parameters. Despite the necessity of taking into account the uncertainties in the modeling and optimization of energy hub, most models proposed for energy hubs in the literature have been scheduled in a deterministic environment. This leads to a reduction in the accuracy of these models and unrealistic results as well as ignoring risks arising from variations in uncertain parameters in optimal decision making. Therefore, future energy hub models need to the realistic modeling of multi-energy systems to be able to achieve a realistic and comprehensive model of future sustainable energy systems.

### REFERENCES


