Solving the Combined Heat and Power Economic Dispatch Problem Considering Power Losses by Applying the Snake Optimization

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The combined heat and power economic dispatch (CHPED) problem seeks to find the optimal point for power and heat generations to minimize the fuel cost considering the problem constraints. In this paper, the snake optimization (SO) algorithm is used to solve the CHPED problem, considering power losses. Two case studies including 5-, and 48-unit test systems have been simulated in MATLAB software. The simulation results of test case 1 verify that the SO reduces the minimum operation costs by at least 0.774%, 0.367%, 0.1437%, 0.143%, 0.1143%, and 0.0215%, compared to the best results of genetic algorithm (GA), harmony search (HS), classic particle swarm optimization (CPSO), imperialist competitive algorithm (ICA), group search optimizer (GSO), and imperialist competitive Harris hawks optimization (ICHHO) algorithms, for load profile 1. It also reduces the minimum operation costs by at least 1.705%, 1.361%, 0.1293%, 1.109%, 0.0957%, and 0.0756%, in compassion to GA, HS, CPSO, ICA, GSO, and ICHHO algorithms for load profile 2. Furthermore, for load profile 3, SO decreases the minimum operation costs by at least 0.5948%, 0.3716%, 0.122%, 0.1206%, and 0.0761% compared to GA, HS, CPSO, ICA, and GSO algorithms. In 48-unit test system, considering power losses, prohibited operating zones, and the valve point loading effect, the reduction of operating costs using the SO algorithm compared to CPSO, gravitational search algorithm (GSA), GA, hybrid time varying acceleration coefficients-GSA-PSO (TVAC-GSA-PSO), group search optimizer (GWO), society-based gray wolf Optimizer (SGWO), and ICHHO algorithms is 1.943%, 1.288%, 0.463%, 0.659%, 0.426%, and 0.197%, respectively.

keywords: *Combined Heat and Power Economic Dispatch, Combined Heat and Power Units, Meta-Heuristic Algorithms, Snake Optimization, Power Losses.*

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NOMENCLATURE

$C_{pi}(P_i)$	The operating cost of the i_{th} power-only unit, (\$/h)
$C_{ci}(O_i, H_i)$	The operating cost of the i_{th} CHP unit, (\$/h)
$C_{hi}(T_i)$	The operating cost of the i_{th} heat-only unit, (\$/h)
H_i/T_i	The output heat energy from the i_{th} heat-only unit/CHP unit, (MWth)
P_i/O_i	The output power from the i_{th} power-only unit/CHP unit, (MW)
$H_i^{min}(o_i), H_i^{max}(o_i)$	The min/max value of output heat energy from the i_{th} unit, (MWth)
$O_i^{min}(H_i), O_i^{max}(H_i)$	The min/max value of output power from the ith unit, (MW)
P_L	Total power losses, (MW)
a_i,b_i,c_i,d_i,e_i	Coefficients of cost function of i_{th} only-power unit
$\eta_i, \theta_i, \lambda_i$	Coefficients of cost function of i_{th} only-heat unit
$a_i, b_i, \gamma_i, \delta_i, \varepsilon_i, \zeta_i$	Coefficients of cost function of i_{th} CHP unit
$B_{i,j}$	Coefficient of power loss between i_{th} and j_{th} units
H_d	Heat energy demand, (MWth)
P_d	Power demand, (MW)

P_i^{min}, P_i^{max} The min/max value of output power from the ith power-only unit, (MW)

T_i^{min}, T_i^{max} The min/max value of output power from the i_{th} heat-only unit, (MWth)

N_p The number of power-only units

N_h The number of heat-only units

N_c The number of CHP units

1. INTRODUCTION

The purpose of the economic dispatch (ED) problem is to find the optimal solution for the operation of the power system in order to minimize the fuel cost while respecting the operational and technical constraints [1], which can be the first attempt of researchers to maximize the benefits. Thermal power plants burn fossil fuels to produce electricity with wasted heat energy [2]. This reduces the efficiency of the energy conversion process in the thermal power plant to about 60% [3]. The integration of cogeneration or CHP units in existing power systems can increase the energy conversion efficiency in the thermal power plant to about 80-90% [4, 5]. CHP units can generate electricity using a variety of fuels and also recover and reuse the heat that is generally lost during electricity generation [6]. cogeneration of electricity and heat from CHP has encouraged many countries and organizations to integrate CHP units into their existing electrical units to improve the sustainability and economic performance of power generation units [7]. On the other hand, power generation by fossil fuels is the main reason for the emission of sulfur dioxide (SO2), nitrogen oxides (NOx), and other greenhouse gases [8]. Reducing greenhouse gas emissions by approximately 13 to 18 percent is one of the other benefits of CHP units. The main objective of the combined heat and power economic dispatch (CHPED) problem is to minimize the operating costs of CHP units, power-only units (POUs), and heat-only units (HOUs) to achieve the maximum profit. The main challenges of the CHPED problem are the valve point loading effect (VPLE) of thermal power units, and heat and power interdependency of CHP units, known as the feasible operating region (FOR). These make CHPED a complex, non-linear and non-convex problem that requires the use of powerful methods to solve it [8].

Over the past two decades, researchers have proposed many solution methods for the CHPED problem. These methods are mainly classified into two categories: classical, or mathematical optimization algorithms and intelligent optimization algorithms.

Classical algorithms include Lagrange Relaxation [9], Lambda Iteration [10], etc. Ideally, these methods give us the best optimal solution for linear cost functions. However, the real-world CHPED problem is nonlinear and non-convex due to the valve point loading effect and prohibited operating zones. Classical optimization algorithms are robust and fast, but their runtimes vary, they are highly sensitive to the initial point of the optimization, and the nature of the objective function. In general, there is no guarantee of obtaining globally or even near-globally optimal solutions with these methods.

Therefore, numerous intelligent, or meta-heuristic algorithms have been employed to solve this problem. Various evolutionary and population-based optimization algorithms have been proposed in the literature for simultaneous economic dispatching of power and heat in the CHP units, e.g., ant colony optimization (ACO) [11], evolutionary programming [12], genetic algorithm (GA) [13], multi-objective particle swarm optimization (MOPSO) [14], strength Pareto honey bee mating optimization (SPHBMO) [15], honey bee colony (BCO) [16], imperialist competitive algorithm (ICA) [17], and cuckoo search algorithm [18]. Some other learning-based intelligent optimization algorithms (LIOA) have also been presented in the literature, like teaching learning-based optimization (TLBO) [19], gravitational search algorithm (GSA) [20], and exchange market algorithm (EMA) [21, 22]. These techniques are derivative-free, and do not require a good starting point. They also cannot guarantee the achievement of the optimal solution and suffer from premature convergence.

To improve the optimization performance, several hybrid and modified algorithms have been proposed by researchers. In [23], the authors have used the real-coded genetic algorithm with random walk-based mutation (RCGA-CRWM), which has improved the results in terms of the cost and convergence speed compared to several methods. A hybrid firefly and self-regulating particle swarm optimization (FSRPSO) has been addressed in [24]. In [25], the improved shuffle frog leaping algorithm (ISFLA) is implemented on a standard test system.. Numerical results show that ISFLA is faster and more accurate than other methods. An improved Niching differential evolution (NDE) is introduced in [26] on a large-scale CHP unit, where the numerical results demonstrate the feasibility and effectiveness of NDE in solving the large-scale CHPED problem. In [27], the society-based Gray Wolf optimizer (SGWO) algorithm is employed by considering the valve point loading effect, prohibited operating zones, and transmission losses.

The power system planners are always looking to reduce various operating costs by using efficient optimization techniques. Therefore, power system optimization, especially in the power generation sector, considering new facilities, such as CHP technology, is an important and interesting task in power system operation. By using more powerful optimization techniques, the cost of power generation will be significantly reduced, and cost savings can be used to develop new projects and replace old equipment with new, and modern ones. The literature review confirms that many optimization algorithms are used to solve this complex problem, including common algorithms such as GA, PSO, ICA, and their different versions. In this regard, the use of new optimization algorithms to reduce the operating cost of various problems such as CHPED is of interest. Here we focus on the Snake optimization algorithm because of its advantages in balancing exploration and exploitation phases, as two important key features in meta-heuristic algorithms.

In general, exploration means ensuring that the search is global, and exploitation denotes finding a more suitable solution around the present one, i.e., local search. In fact, the main challenge is to balance these two factors. Therefore, it is necessary to use the appropriate compromise between exploration and exploitation phases to find the optimal solutions. In order to deal with the shortcomings of classical algorithms in solving non-linear and non-convex problems, as well as reducing the complexity of hybrid algorithms, this paper uses a new meta-heuristic algorithm called snake optimization (SO) [28] to solve the CHPED problem. Also, the results obtained are compared with other optimization algorithms presented in the literature. Two case studies of 5-, and 48-unit test systems are considered to implement the SO algorithm.

The main novelties of the paper are:

- Applying the SO algorithm to solve the CHPED problem for the first time
- Comparison of the optimal operating costs obtained by

the SO algorithm for two test cases, with other methods available in the literature

- Proposing new constraint handling method for solving the CHPED problem.

The rest of the paper is organized as follows. Section 2 describes the problem formulation and also the SO algorithm. The simulation results are presented in Section 3, where a comparative study is conducted to confirm the superiority of the SO algorithm. Finally, Section 4 concludes the paper.

2. MATHEMATICAL FORMULATION AND OPTIMIZATION METHOD

The CHPED is one of the critical topics in power systems. In this problem, we deal with power-only units, heat-only units, and combined heat and power generation units. In this section the mathematical formulation of the CHPED problem, and the optimization algorithm used, i.e., SO, are explained in detail.

A. The Problem Formulation

The purpose of the CHPED problem is to minimize the fuel costs associated with the combined power and heat, power-only and heat-only units so that all the constraints and heat and power demands are met. The general mathematical structure of this problem is as following [27, 29, 30]:

$$min \;\; OF = \sum_{i=1}^{N_p} C_{pi}(P_i) + \sum_{i=1}^{N_c} C_{ci}(O_i, H_i) + \sum_{i=1}^{N_h} C_{hi}(T_i) \tag{1}$$

$$O_i^{\min}(H_i) \leq O_i \leq O_i^{\max}(H_i) \tag{2}$$

$$H_i^{\min}(O_i) \leq H_i \leq H_i^{\max}(O_i) \tag{3}$$

$$P_i^{\min} \leq P_i \leq P_i^{\max} \;\; i = 1, ..., N_p \tag{4}$$

$$T_i^{\min} \leq T_i \leq T_i^{\max} \;\; i = 1, ..., N_T \tag{5}$$

$$\sum_{i=1}^{N_p} P_i + \sum_{i=1}^{N_c} O_i = P_d \tag{6}$$

$$\sum_{i=1}^{N_c} H_i + \sum_{i=1}^{N_h} T_i = H_d \tag{7}$$

$$\sum_{i=1}^{N_p} P_i + \sum_{i=1}^{N_c} O_i = P_d + P_L \tag{8}$$

Where

$$C_{pi}(P_i) = a_i P_i{}^2 + b_i P_i + c_i + |d_i sin\{e_i(P_i^{\min} - P_i\}| \tag{9}$$

$$C_{ci}(O_i, H_i) = a_i O_i^2 + \beta_i O_i + \gamma_i + \delta_i H_i^2 + \varepsilon_i H_i + \zeta_i O_i H_i \tag{10}$$

$$C_{hi}(T_i) = \eta_i T_i{}^2 + \theta_i T_i + \lambda_i \tag{11}$$

$$P_L = \textstyle\sum_{i=1}^{N_p} \displaystyle\sum_{j=1}^{N_p} P_i B_{ij} P_j + \textstyle\sum_{i=1}^{N_p} \displaystyle\sum_{j=1}^{N_c} P_i B_{ij} O_j + \textstyle\sum_{i=1}^{N_p} \displaystyle\sum_{j=1}^{N_c} O_i B_{ij} O_j \tag{12}$$

$$P_i \in \begin{cases} P_i^{\min} \leq P_i \leq P_{i,1}^{l} \\ P_{i,m-1}^{U} \leq P_i \leq P_{i,m}^{l} & m = 2, 3, ..., Z_i \\ P_{i,Zi}^{U} \leq P_i \leq P_i^{\max} \end{cases} \tag{13}$$

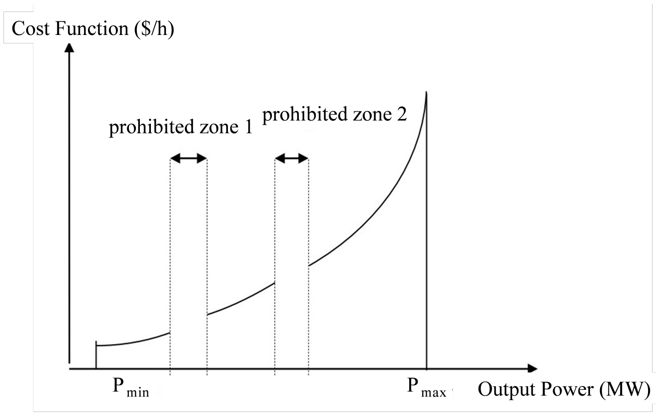

Fig. 1. Prohibited operating zones for power-only units

The first equation demonstrates the objective function of the CHPED problem. Eqs (2)-(5) indicate the inequality constraints related to CHP, POUs, and HOUs, respectively. The power and heat balances are mentioned in Eqs. (6)-(7). Eq. (8), considers the power losses in power balance. The cost functions of the CHP, POUs, and HOUs are illustrated in Eqs (9)-(11).

It is worth mentioning that in Eq. (9), the term $d_i * sin(e_i(P_i^{min} - P_i))$ denotes the valve point loading effect, which creates a non-convex objective function. Eq. (13) indicates the prohibited operating zones constraints, which converts the cost function in Eq. (9) into a discontinuous curve, as shown in Fig. 1. In this study, Kron's loss formula based on Eq. (12), is used to calculate the power losses.

B. Optimization algorithm

In this section, the model of the snake optimization (SO) algorithm is presented. Interested readers are referred to [28] for more details. Mating of snakes occurs when the temperature (Temp) is low and food (Q) is available, otherwise snakes just forage or eat what is available. Parameters Temp and Q can be defined as follows:

$$Temp = \exp(\tfrac{-t}{T}), Q = c_1 \times \exp(\tfrac{t-T}{T}) \tag{14}$$

where, t is the current iteration and T refers to the maximum number of iterations. Also, c1 is a constant. The optimization process is divided into two phases, i.e., exploration and exploitation. Like all meta-heuristic algorithms, SO starts by generating a random population with a uniform distribution. The initial population can be obtained using Eq. (15).

$$X_i = X_{\min} + r \times (X_{\max} - X_{\min}) \tag{15}$$

where, X_i is the individual position, r is a random number between 0 and 1, and X_{min} and X_{max} are the lower and upper bounds of the population, respectively. The population is divided into two groups, i.e., male and female. Eq. (16) is used for this purpose.

$$N_m \approx \tfrac{N}{2}, N_f = N - N_m \tag{16}$$

where, N is the number of the initial population, N_m and N_f refer to the number of male and female snakes, respectively. If $Q < Q_{critical}$, snakes take the exploration phase, which means they search for food by choosing any random position, and update their position accordingly. The exploration phase formulation is as follows:

$$X_i(t+1) = X_{rand}(t) \pm c_2 \times A \times ((X_{\max} - X_{\min}) \times rand + X_{\min}) \tag{17}$$

where, X_i refers to the position of the i_{th} male/female snake, X_{rand} denotes the position of the random male/female snake, rand is a random number between 0 and 1, c_2 is a constant, and $A$is the ability of the male/female snake to find food, as follows:

$$A = \exp(\frac{-f_{rand}}{f_i}) \tag{18}$$

where, f_{rand} is the fitness function associated with X_{rand}, and f_i is the fitness function of the i_{th} member of the male/female group. If $Q > Q_{critical}$ and $Temp < Temp_{critical}$, snakes take the exploitation phase. In other word, the mating occurs. The exploitation phase formulation is as follows: where X_i is the position of the i_{th} male/females snake, c_3 is a constant, and M refer to the mating ability of male/female snake, which can be defined as follows:

$$X_i(t+1) = X_i(t) \pm c_3 \times M \times rand \times (Q \times \left| X_{i,f}(t) - X_{i,m}(t) \right|) \tag{19}$$

where X_i is the position of the i_{th} male/females snake, c_3 is a constant, and M refer to the mating ability of male/female snake, which can be defined as follows:

$$M_m = \exp(\frac{-f_{i,f}}{f_{i,m}}), \quad M_f = \exp(\frac{-f_{i,m}}{f_{i,f}}) \tag{20}$$

Selecting the initial parameters of the SO algorithm affect the convergence speed of the algorithm. The initial parameters of the SO are: the maximum number of iterations (T), the number of initial population (N), the threshold of food ($Q_{critical}$), the threshold on temperature ($Temp_{critical}$), and the constant coefficients of c_1, c_2 and c_3. The optimization procedure for the CHPED problem is summarized as Algorithm 1. Finally, the flowchart of the algorithm is shown in Fig. 2.

C. Constraint handling method

To obtain higher quality solutions, it is necessary to use an effective method to control the constraints. In this work, a new method is used to satisfy the constraints. Generally, two types of methods are considered to satisfy the constraints, namely, penalty factor and constraint repair step [31]. By using the penalty factor method, the algorithm concentrates on handling the constraints instead of obtaining better results. Unlike the previous method, applying the constraint repair step leads to high quality results as well as handling the constraints [32]. In another word, it increases the accuracy and quality of the results. In this paper, a new method is used to satisfy the equality constraints (Eqs (6), (8)) based on the constraint repair step method. In this way, the total difference of power and heat produced from the power and thermal demands is allocated to the unit, which leads to the least increase in cost or the most cost reduction.

$$\Delta P = P_d + P_L - (\sum_{i=1}^{N_P} P_i + \sum_{i=1}^{Nc} O_i) \tag{21}$$

Eq. (21) demonstrates the difference between demand power and generated power, it is expected that ΔP be a number close to zero. In this work, this number is considered 0.0001 to ensure the balance of generated and demand powers, and if ΔP is greater than 0.0001, then ΔP is assigned to the POU with the least cost increase or the most cost reduction. For this purpose.

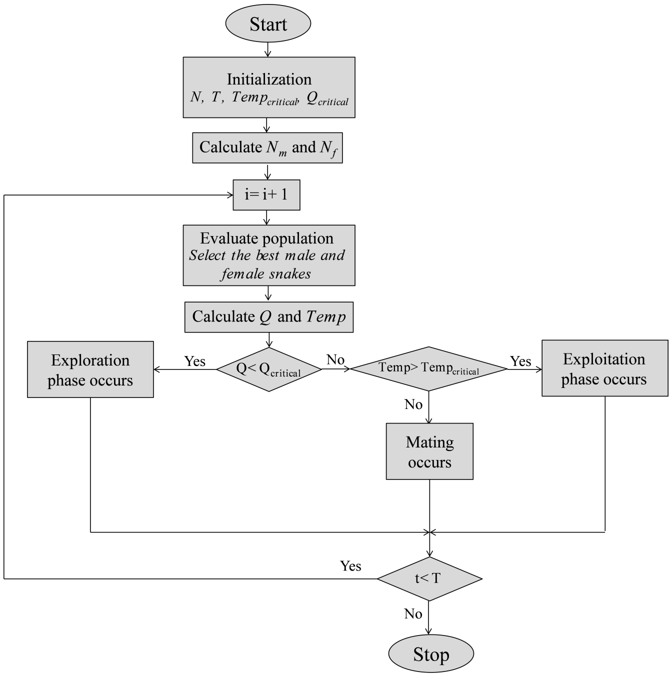

Fig. 2. The flowchart of the SO algorithm

- if ΔP is a positive value greater than 0.0001, it means that more power must be generated. The new generation cost for each of the POUs is calculated using Eq. (22), and the POUs with the lowest cost increase are prioritized using Eq. (23). The power of ith unit is increased, which has a lower cost increase.
- If ΔP is a negative value less than -0.0001, it means that less power must be generated. The new generation cost for each of the POUs is calculated using Eq. (22), and the POUs with the highest cost decrease are prioritized to reduce their generated power (Eq. 23).

$$C_{pi}(P_i + \Delta P) = a_i(P_i + \Delta P)^2 + b_i(P_i + \Delta P) + c_i + |d_i sin\{e_i(P_i^{\min} - (P_i + \Delta P))\}| \tag{22}$$

$$\Delta C_{pi} = \left| C_{pi}\ (P_i + \Delta P) - C_{pi}\ (P_i) \right| \tag{23}$$

Similar approach is applied to HOUs, using Eqs. (24)-(26).

$$\Delta H = H_d - (\sum_{i=1}^{N_c} H_i + \sum_{i=1}^{N_h} T_i) \tag{24}$$

$$C_{hi}(T_i + \Delta H) = \eta_i(T_i + \Delta H)^2 + \theta_i(T_i + \Delta H) + \lambda_i \tag{25}$$

$$\Delta C_{hi} = |C_{hi}\ (T_i + \Delta H) - C_{hi}\ (T_i)| \tag{26}$$

3. SIMULATION RESULTS

In order to evaluate the performance of the SO algorithm, the CHPED problem is solved for two test systems and the results are compared with other optimization algorithms *[1]. Detailed data of two case studies are provided in Appendix A. The parameters of the SO algorithm for each of the test systems are given in Table 1.

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[1] The authors are ready to share the MATLAB codes with those who wish to replicate the results. Please contact the first author.

Table 1. Algorithm 1. Snake optimization algorithm

Algorithm 1. Snake optimization algorithm
1. Selecting the initial parameters of the algorithm
2. Setting the number of decision variables according to the number of units in the CHP system.
The decision variables for the CHPED problem are the power and heat generation capacity of the power-only, heat-only and CHP units
3. Setting the upper and lower limits of the power and heat generations.
4. Calculating the amount of power and heat generation based on the position of the snakes and choosing the best snake.
5. Updating the position of the snakes in each iteration using Eqs. (17) or (19) according to the exploration and explotation phase.
6. Selecting the snake with the lowest fitness (objective) function and printing the position of the best snake associated with the lowest cost.

Table 2. Parameters of SO algorithm for different case studies

Parameter	Test system 1	Test system 2
Maximum iterations	400	250
Number of snakes	500	750
$Q_{critical}$	0.25	0.25
$Temp_{critical}$	0.7	0.625
c1, c2, c3	0.35, 0.1, 2	0.55, 0.05, 2.4

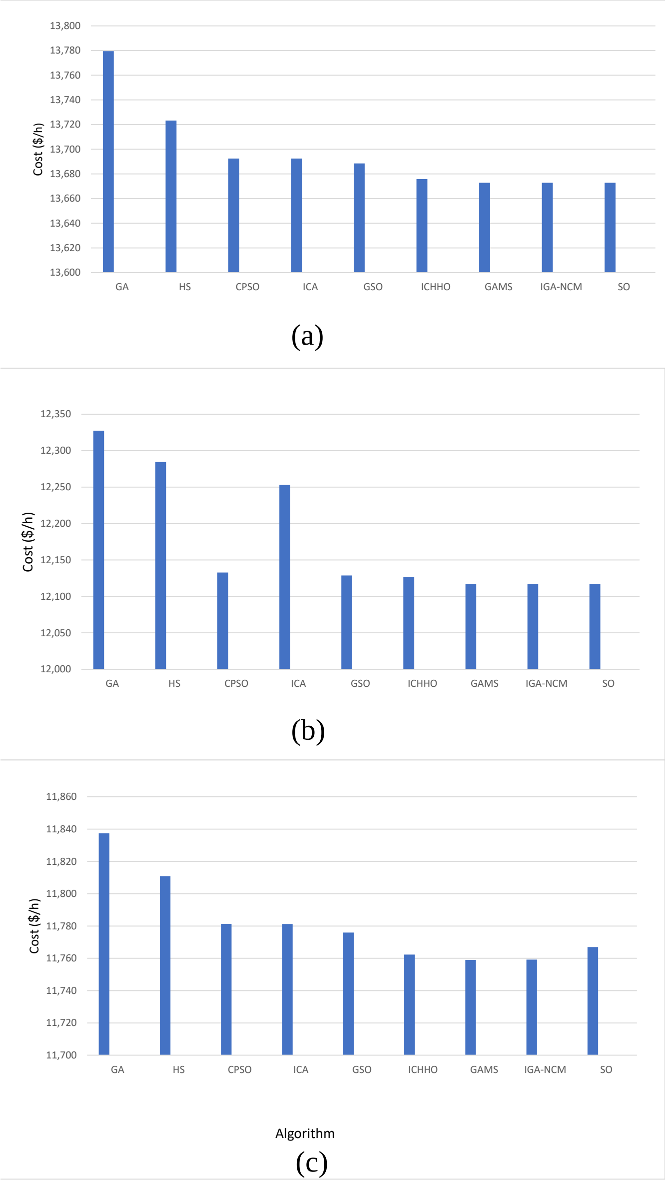
A. Test system 1

Test system 1 includes one POU, three CHP, and one HOU. Power losses are not considered in this case study. The system includes three different load profiles as Table 2. The obtained results are compared with the other algorithms proposed in the literature in Table 3. Furthermore, Fig. 3 compares the total costs obtained by employing different algorithms for three load profiles.

As seen in Table 3 and Fig. 3(a), the obtained results prove the superiority of the SO algorithm compared to other algorithms for load profile 1. So, by supposing the constant annual load, the annual cost savings of the SO compared to GA, HS, CPSO, ICA, GSO, and ICHHO algorithms are $934,396, $441,208, $172,462, $171,568, $137,093, and $25,824, respectively. The results obtained from the SO algorithm, as shown in Table 3 and Fig. 3(b) for load profile 2, show the superiority of the solutions compared to other algorithms. The annual cost savings of the SO compared to GA, HS, CPSO, ICA, GSO, and ICHHO algorithms are equal to $1,841,353, $1,465,374, $137,427, $1,190,753, $101,709, and $80,397, respectively. In load profile 3, the SO algorithm obtains a more optimal solution compared to the GA, HS, CPSO, ICA, and GSO algorithms, as shown in Table 3 and Fig. 3(c). The annual cost savings of the SO compared to GA, HS, CPSO, ICA, and GSO algorithms will be $616,824, $384,508, $125,992, $124,533, and $78,508, respectively

B. Test system 2

In this subsection, a 48-unit system that includes 26 power-only, 12 CHP, and 10 heat-only units [20] is considered as the second case study, where power and heat demands are 4700 MW and 2500 MWth, respectively (see appendix A for more details). The convergence curve associated with the cost function is depicted in Fig. 4. The numerical and comparative results with other methods are presented in Table 4. Also, the total cost of different



(a)

(b)

(c)

Fig. 3. Total cost obtained by different algorithms for test system 1, (a) load profile 1, (b) load profile 2, (c) load profile 3

Table 3. Load profiles of the test system 1

Test system	Load profile 1		Load profile 2		Load profile 3	
	Power (MW)	Heat (MWth)	Power (MW)	Heat (MWth)	Power (MW)	Heat (MWth)
Test system 1	300	150	250	175	160	220

Table 4. Comparison of results obtained by using different algorithms for test system 1

Method	P1 (MW)	O1 (MW)	O2 (MW)	O3 (MW)	H1 (MWth)	H2 (MWth)	H3 (MWth)	T1 (MWth)	Total cost ($/h)
Load Profile 1									
GA [5]	135.0000	70.8100	10.8400	83.2800	80.5400	39.8100	0.0000	29.6400	13,779.5000
HS [5]	134.7400	48.2000	16.2300	100.8500	81.0900	23.9200	6.2900	38.7000	13,723.2000
CPSO [33]	135.000	40.7309	19.2728	105.0000	64.4003	26.4119	0.0000	59.1955	13,692.5212
ICA [34]	134.9963	40.7309	19.2728	105.0000	64.4003	26.4119	0.0000	59.1878	13,692.4191
GSO [35]	135.0000	40.7214	19.4673	104.8111	66.5830	26.8103	0.0378	56.5687	13,688.4836
ICHHO [36]	134.9839	40.1606	20.3595	104.4870	72.6278	36.2074	0.0000	41.1548	13,675.7817
GAMS [37]	135.0000	40.7689	19.2311	105.0000	73.59553	36.77661	0.0000	39.62785	13672.83413
Improved GA with novel crossover and mutation (IGA-NCM) [38]	135.0000	40.7689	19.2311	105.0000	73.5955	36.7766	0.0000	39.6278	13672.8341
SO	135.0000	40.7763	19.2236	105	73.5982	36.7600	0	39.6416	13672.8337
Load Profile 2									
GA [5]	119.2200	45.1200	15.8200	69.8900	78.9400	22.6300	18.4000	54.9900	12,327.3700
HS [5]	134.6700	52.9900	10.1100	52.2300	85.6900	39.7300	4.1800	45.4000	12,284.4500
CPSO [33]	135.0000	40.3446	10.0506	64.6060	70.9318	39.9918	4.0773	60.0000	12,132.8579
ICA [34]	129.7710	40.4355	14.0021	65.7911	75.1881	27.3526	22.3190	50.1401	12,253.1006
GSO [35]	134.9953	40.2832	10.0962	64.6251	71.7131	39.8592	6.1571	57.2704	12,128.7805
ICHHO [36]	135.0000	40.1402	10.2953	64.5546	72.9291	38.8186	24.0711	39.1713	12,126.3476
GAMS [37]	135.000	40.0000	10.0000	65.0000	75.0000	40.0000	14.05948	45.94052	12,117.17012
IGA-NCM [38]	135.000	40.0000	10.0000	65.0000	75.0000	40.0000	14.0595	45.9405	12,117.1701
SO	135.0000	40.0000	10.0000	65.0000	75.0000	40.0000	14.0577	45.9422	12117.16981
Load Profile 3									
GA [5]	37.9800	76.3900	10.4100	35.0300	106.0000	38.3700	15.8400	59.9700	11,837.4000
HS [5]	41.4100	66.6100	10.5900	41.3900	97.7300	40.2300	22.8300	59.2100	11,810.8800
CPSO [33]	35.5972	57.3554	10.0070	57.0587	89.9767	40.0025	30.0232	60.0000	11,781.3690
ICA [34]	35.5789	57.3554	10.0070	57.0587	89.9767	40.0025	30.0232	59.9976	11,781.2024
GSO [35]	43.1882	66.4693	10.0492	40.2931	97.7766	39.9777	22.3895	59.8561	11,775.9485
ICHHO [36]	40.0839	62.4169	10.0069	47.4823	94.3513	39.9722	25.6738	59.9927	11,762.3265
GAMS [37]	42.18183	64.6699	10.0000	43.14827	96.29624	40.0000	23.70376	60.0000	11759.00968
IGA-NCM [38]	42.5986	65.1343	10.0058	42.2613	96.6971	40.0025	23.3005	59.9999	11759.1827
SO	42.0631	65.0291	10.0000	42.9076	96.4063	40	23.5944	60.0000	11766.9863

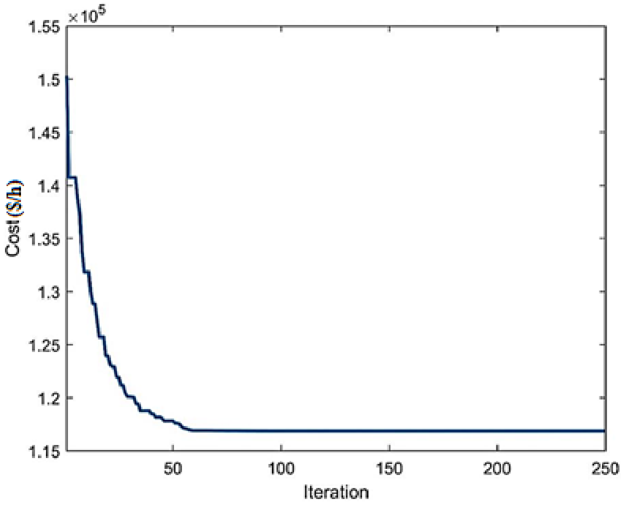


Fig. 4. The convergence curve for test system 2

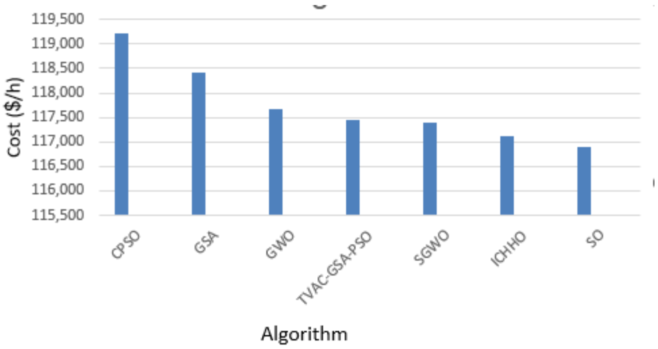


Fig. 5. Total cost obtained by various algorithms for test system 2

algorithms for this test system is given in Table 5. In addition, Fig. 5 demonstrates the total cost derived by applying various algorithms for test system 3.

As can be seen in Tables 4-5, and Figs. 4-5, the obtained results demonstrate the superiority of the SO algorithm over other algorithms in terms of the total cost function. Assuming constant annual load, the annual cost savings associated with the SO algorithm compared to CPSO, GSA, GWO, TVAC-GSA-PSO, SGWO, and ICHHO algorithms will be \$20,292,258, \$1,3368,266, \$6800,670, \$4764,655, \$4,390,565, and \$2,030,210, respectively.

C. Validation of results

In this section, the obtained results are validated in terms of different equality constraints of the problem. For this purpose, power and heat balance violations (in MW, MWth) for each test system are presented. Equations (6), and (7) are used for test system 1, and equations (8), and (7) are checked for test system 2. Tables 6, and 7 show power and heat balance violations for each test system. As tables 6 and 7 show, the equality constraints of power and heat balance for both systems have been fully satisfied by using SO algorithm, and none of these two important constraints have been violated. This validates the correctness of the SO algorithm in the obtained results.

4. CONCLUSION

The CHPED problem is a challenging issue in power system operation that has attracted the attention of many researchers. In this paper, SO algorithm as a powerful meta-heuristic algorithm is used for the first time, to solve the mentioned problem. To evaluate the performance of the SO algorithm, two test systems including a small-scale and a large-scale, are simulated. The results show the superiority of the SO algorithm over the different algorithms reported in the literature. Numerical results show that the SO algorithm has the least total cost compared to other algorithms for test system 1. Specifically, the reduction of operating costs using the SO algorithm compared to GA, HS, CPSO, ICA, GSO, and ICHHO algorithms is 0.774%, 0.367%, 0.1437%, 0.143%, 0.1143%, and 0.0215%, for load profile 1, respectively. It saves 106.67,50.37, 19.69,19.59, \$15.65, and 2.95\$ per hour. The results obtained are similar compared to GAMS, and IGA-NCM technique. The reduction of operating costs for load profile 2, using the SO algorithm compared to GA, HS, CPSO, ICA, GSO, and ICHHO algorithms is 1.705%, 1.361%, 0.1293%, 1.109%, 0.0957%, and 0.0756%, respectively. It saves \$210.2, \$167.28, \$15.69, \$135.93, \$11.61, and 9.18\$ per hour. The results obtained are similar compared to GAMS, and IGA-NCM technique. Also, for load profile 3, the reduction of operating costs, using the SO algorithm compared to GA, HS, CPSO, ICA, and GSO algorithms is 0.5948%, 0.3716%, 0.122%, 0.1206%, and 0.0761% respectively. It saves \$70.41, \$46.89, \$14.38, \$14.22, and \$8.96 per hour, respectively. The results obtained are not better compared to ICHHO, GAMS, and IGA-NCM technique. In large-scale systems, a 48-unit test system is considered, and to make the modeling closer to reality, power losses and prohibited operating zones are included in the system in addition to the valve point loading effect. In this system, considering losses, the reduction of operating costs using the SO algorithm compared to CPSO, GSA, GA, TVAC-GSA-PSO, GWO, SGWO, and ICHHO algorithms is 1.943%, 1.288%, 0.463%, 0.659%, 0.426%, and 0.197%, respectively. It saves \$2316.468, \$1526.06, \$543.9104, \$776.332, \$501.2061, and \$231.7592 per hour, respectively. Also, the SO algorithm reduces the total losses compared to CPSO, GSA, GA, TVAC-GSA-PSO, GWO, SGWO, and ICHHO algorithms by 8.571%, 8.628%, 8.499%, 8.492%, 8.490%, and 11.992% respectively. Considering the total cost of the SO algorithm for large-scale systems, it is evident that SO algorithm has an acceptable performance. The use of the mentioned algorithm to solve the multi-objective CHPED problem can be considered in future research. Also, problem solving in the presence of different uncertainties can be focused in this direction

5. APPENDIX A

Appendix A.1. Feasible Operation Regions of CHP Units [39]

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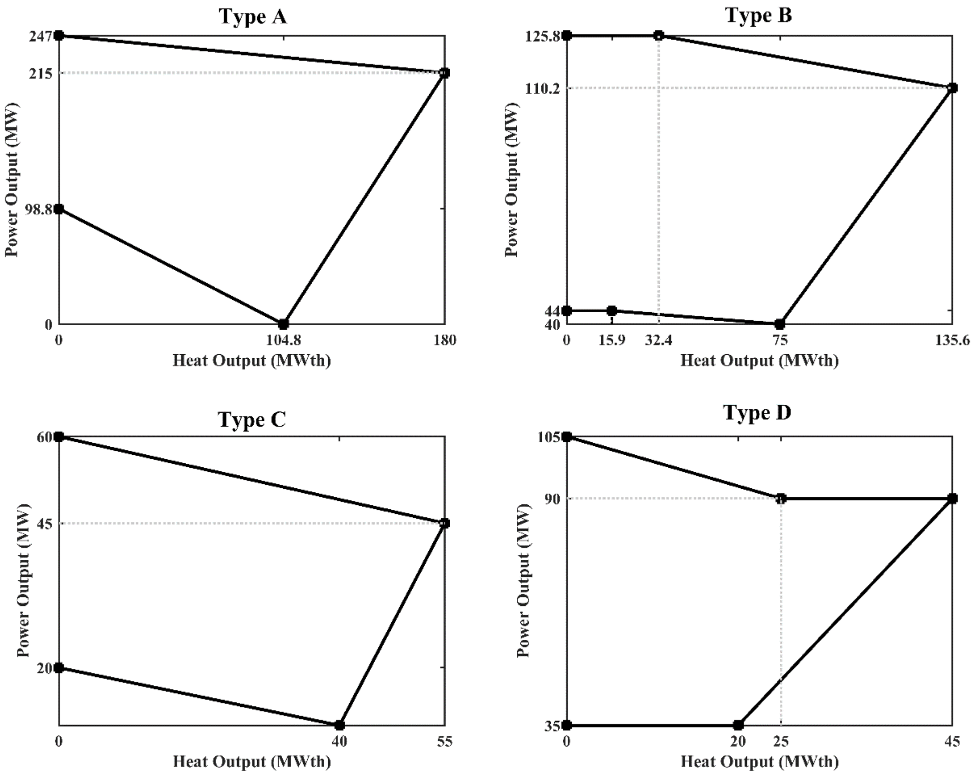
Table 5. Comparison of results obtained by using conventional algorithms for test system 2

Output \Algorithm	CPSO [29]	GSA [29]	TVAC-GSA-PSO [29]	GWO [27]	SGWO [27]	ICHHO [36]	SO
P1	429.3029	459.2381	448.8078	447.8419	448.7562	628.9271	538.5670
P2	156.9660	151.3582	149.6312	148.3678	149.5853	302.3341	299.1985
P3	359.9999	226.5440	299.2780	296.7675	299.2117	292.3650	299.1988
P4	114.0122	110.7530	109.9978	109.4606	109.9606	158.4509	60
P5	119.4656	109.0544	109.8386	110.3213	109.7674	109.8177	60
P6	62.6231	61.2220	60.0024	60.0027	60.0010	60.0593	60
P7	159.8864	164.1648	160.1809	159.5677	160.1776	111.1994	60
P8	60.0154	60.0000	60.0054	60.0041	60.0037	110.0143	159.7329
P9	156.8785	164.3867	159.9778	160.4262	159.7020	159.9923	159.7329
P10	119.9201	114.8650	114.5007	114.9654	114.4622	40.6526	40
P11	78.2515	79.9657	78.1772	76.2326	77.3884	40.4774	40
P12	97.2142	99.9484	92.0908	91.8974	92.3198	55.2024	55
P13	96.8773	95.3491	92.4547	92.1564	92.4549	55.0943	55
P14	383.8960	356.8072	359.0720	355.8358	359.0301	630.2608	563.2682
P15	224.3990	360.0000	356.6475	356.1114	359.2257	300.6216	299.1988
P16	359.9349	359.9999	359.9883	359.7121	359.9515	301.2961	299.1988
P17	153.7382	167.9340	159.7407	159.4746	159.6778	107.6883	159.7329
P18	60.0726	60.0000	60.0062	60.0402	60.0024	60.04420	159.7329
P19	158.0583	161.4781	159.7182	159.8005	159.6795	109.6910	159.7329
P20	165.9378	164.4771	159.8033	160.4356	159.7446	60.0250	159.7329
P21	162.7038	156.1363	159.7097	168.3596	159.7375	159.9303	159.7329
P22	162.2120	156.8002	159.4594	159.9232	159.7382	60.0757	159.7329
P23	79.9436	119.9992	78.6893	75.0542	77.4043	40.1045	40
P24	41.3818	40.0001	40.0010	40.0000	40.0000	76.9169	40
P25	91.7993	91.7586	92.1903	92.1314	92.2106	55.0792	55
P26	91.2537	89.3618	92.4586	92.6610	92.5018	92.3888	55
O1	123.9223	107.9560	114.1312	117.7696	113.9997	101.3603	81
O2	40.9141	40.1911	40.0024	40.0000	39.9998	41.1043	40
O3	84.8033	81.0020	81.0382	81.0026	81.1164	88.0631	95.7512
O4	50.4196	54.7271	54.2485	54.5421	54.2483	42.1218	40
O5	10.0012	10.5290	10.0000	10.0019	10.0011	10.2108	10
O6	36.7773	35.2482	35.0544	35.0200	35.0483	41.1555	38.3296
O7	89.6612	84.4300	86.6357	86.8096	86.6102	83.8499	95.9700
O8	40.2019	43.3853	41.6388	41.8154	41.5933	40.4689	40
O9	99.5361	82.8585	86.1925	86.9121	86.0666	91.6621	81
O10	46.9061	48.7236	49.4750	49.4124	49.4499	48.9416	40
O11	10.0670	10.3059	10.0023	10.0012	10.0001	10.3403	10.00003
O12	35.9853	35.0533	35.0022	35.0019	35.0085	42.4462	37.4577
H1	127.1189	119.6859	123.3821	125.3161	123.2964	116.2260	104.8000
H2	75.7733	72.1576	74.9844	74.9845	74.9844	75.9533	75

Output \Algorithm	CPSO [29]	GSA [29]	TVAC-GSA-PSO [29]	GWO [27]	SGWO [27]	ICHHO [36]	SO
H3	106.7115	104.7912	104.8060	104.7879	104.8554	108.7638	113.0783
H4	83.9222	84.6009	87.2750	87.4892	87.2786	76.8133	75
H5	40.0005	40.2267	40.0000	40.0005	39.9957	40.0872	40
H6	20.7961	20.1128	20.0184	19.9988	20.0211	22.7979	21.5134
H7	105.4107	106.7149	107.9526	107.9254	107.9382	106.3993	113.2011
H8	72.1171	77.9056	76.3959	76.5348	76.3593	75.4048	75
H9	110.0942	105.8329	107.6974	108.0893	107.6333	110.7835	104.8000
H10	77.8780	78.9748	83.1570	82.9662	83.1253	82.7188	75
H11	40.0085	40.1311	39.9959	40.0005	39.9997	40.1458	40.00001
H12	19.8703	20.0242	19.9999	20.0009	20.0039	23.3846	21.1171
T1	429.7405	455.7303	455.3517	455.0352	455.4105	448.3868	459.8592
T2	59.9966	60.0000	59.9990	59.9942	60.0000	60.0000	60
T3	59.0421	60.0000	60.0000	59.9013	59.9932	60.0000	60
T4	117.9232	120.0000	119.9998	119.9497	119.9819	120.0000	120
T5	119.7669	120.0000	119.9965	119.9991	120.0000	120.0000	120
T6	479.7902	455.6580	439.0752	437.0706	439.1973	452.1342	461.6306
T7	58.1170	57.4532	59.9982	59.9935	60.0000	60.0000	60
T8	59.7238	60.0000	59.9982	59.9866	60.0000	60.0000	60
T9	120.0000	120.0000	119.9999	119.9757	120.0000	120.0000	120
T10	116.1983	120.0000	119.9168	120.0000	119.9258	120.0000	120
Total losses (MW)	115.940	116.012	115.849	115.8400	115.8370	120.4471	106.0024
Total cost ($/h)	119211.1607	118420.7506	117438.6032	117671.0251	117395.8989	117126.452	116894.6928



Appendix A.1. Feasible Operation Regions of CHP Units [39]

Table 6. Comparing the total cost of various algorithms for test system 2

Algorithm	Total cost ($/h)
CPSO [29]	119,211.1607
GSA [29]	118,420.7506
GWO [27]	117,671.0251
TVAC-GSA-PSO [29]	117,438.6032
SGWO [27]	117,395.8989
ICHHO [36]	117,126.4520
SO	116,894.6928

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Table 7. Power and heat balance violations for test system 1

Method	Total power (MW)	Total heat (MWth)	Power balance violation (MW)	Heat balance violation (MWth)
		Load Profile 1		
GA [5]	299.93	149.99	-0.07	-0.01
HS [5]	300.02	150	0.02	0
CPSO [33]	300.0037	150.0077	0.0037	0.0077
ICA [34]	300	150	0	0
GSO [35]	299.9998	149.9998	-0.0002	-0.0002
ICHHO [36]	299.991	149.99	-0.009	-0.01
GAMS [37]	300	149.9999	0	-1/00E-05
IGA-NCM [38]	300	149.9999	0	-0.0001
SO	299.9999	149.9998	0.0	-0.0002
		Load Profile 2		
GA [5]	250.05	174.96	0.05	-0.04
HS [5]	250	175	0	0
CPSO [33]	250.0012	175.0009	0.0012	0.0009
ICA [34]	249.9997	174.9998	-0.0003	-0.0002
GSO [35]	249.9998	174.9998	-0.0002	-0.0002
ICHHO [36]	249.9901	174.9901	-0.0099	-0.0099
GAMS [37]	250	175	0	0
IGA-NCM [38]	250	175	0	0
SO	250	174.9999	0.0	0.0
		Load Profile 3		
GA [5]	159.81	220.18	-0.19	0.18
HS [5]	160	220	0	0
CPSO [33]	160.0183	220.0024	0.0183	0.0024
ICA [34]	160	220	0	0
GSO [35]	159.9998	219.9999	-0.0002	-0.0001
ICHHO [36]	159.99	219.99	-0.01	-0.01
GAMS [37]	160	220	0	0
IGA-NCM [38]	160	220	0	0
SO	160	220	0.0	0.0

Table 8. Power and heat balance violations for test system 2

Method	Total power (MW)	Total heat (MWth)	Total losses (MW)	Power balance violation (MW)	Heat balance violation (MWth)
CPSO [29]	4815.9395	2499.9999	115.940	0.0	0.0
GSA [29]	4816.0119	2500.0001	116.012	0.0	0.0
TVAC-GSA-PSO [29]	4815.849	2500.0001	115.849	0.0	0.0
GWO [27]	4815.84	2500	115.8400	0.0	0.0
SGWO [27]	4815.837	2500	115.8370	0.0	0.0
ICHHO [36]	4820.434	2499.9993	120.4471	-0.0131	0.0
SO	4806.00183	2499.99971	106.0024	0.0	0.0

Appendix A.2. 5-Unit Test System Data [20]

Unit No.	Cost Function	Capacity or FOR
1	$C_{p1} = 0.000115{P_1}^3 + 0.00172P^2{}_1 + 7.699P_1 + 254.8863$	$35 \leq P_1 \leq 135MW$
2	$C_{C1}(O_1, H_1) = 0.0435{O_1}^2 + 36O_1 + 0.027{H_1}^2 + 0.6H_1 + 0.011O_1H_1 + 1250$	CHP type B
3	$C_{C2}(O_2, H_2) = 0.1035{O_2}^2 + 34.5O_2 + 0.025{H_2}^2 + 2.203H_2 + 0.051O_2H_2 + 2650$	CHP type C
4	$C_{C3}(O_3, H_3) = 0.072{O_2}^2 + 20O_3 + 0.02{H_3}^2 + 2.34H_3 + 0.04O_3H_3 + 1565$	CHP type D
5	$C_{h1}(T_1) = 0.038{T_1}^2 + 2.0109T_1 + 950$	$0 \leq T_1 \leq 60MWth$

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Appendix A.3. 48-Unit Test System Data [20]

Unit No.	Cost Function	Capacity or FOR
1	$C_{p1}(P_1) = 0.00028{P_1}^2 + 8.1P_1 + 550 + \lvert 300sin(0.035\times(-P_1))\rvert$	$0 \leq P_1 \leq 680$
2	$C_{p2}(P_2) = 0.00056{P_2}^2 + 8.1P_2 + 309 + \lvert 200sin(0.042\times(-P_2))\rvert$	$0 \leq P_2 \leq 360$
3	$C_{p3}(P_3) = 0.00056{P_3}^2 + 8.1P_2 + 309 + \lvert 200sin(0.042\times(-P_3))\rvert$	$0 \leq P_3 \leq 360$
4	$C_{p4}(P_4) = 0.00324{P_4}^2 + 7.74P_4 + 240 + \lvert 150sin(0.063\times(60 - P_4))\rvert$	$0 \leq P_4 \leq 180$
5	$C_{p5}(P_5) = 0.00324{P_5}^2 + 7.74P_5 + 240 + \lvert 150sin(0.063\times(60 - P_5))\rvert$	$0 \leq P_5 \leq 180$
6	$C_{p6}(P_6) = 0.00324{P_6}^2 + 7.74P_6 + 240 + \lvert 150sin(0.063\times(60 - P_6))\rvert$	$0 \leq P_6 \leq 180$
7	$C_{p7}(P_7) = 0.00324{P_7}^2 + 7.74P_7 + 240 + \lvert 150sin(0.063\times(60 - P_7))\rvert$	$0 \leq P_7 \leq 180$
8	$C_{p8}(P_8) = 0.00324{P_8}^2 + 7.74P_8 + 240 + \lvert 150sin(0.063\times(60 - P_8))\rvert$	$0 \leq P_8 \leq 180$
9	$C_{p9}(P_9) = 0.00324{P_9}^2 + 7.74P_9 + 240 + \lvert 150sin(0.063\times(60 - P_9))\rvert$	$0 \leq P_9 \leq 180$
10	$C_{p10}(P_{10}) = 0.00284{P_{10}}^2 + 8.6P_{10} + 126 + \lvert 100sin(0.084\times(40 - P_{10}))\rvert$	$40 \leq P_{10} \leq 120$
11	$C_{p11}(P_{11}) = 0.00284{P_{11}}^2 + 8.6P_{11} + 126 + \lvert 100sin(0.084\times(40 - P_{11}))\rvert$	$40 \leq P_{11} \leq 120$
12	$C_{p12}(P_{12}) = 0.00284{P_{12}}^2 + 8.6P_{12} + 126 + \lvert 100sin(0.084\times(40 - P_{12}))\rvert$	$55 \leq P_{12} \leq 120$
13	$C_{p13}(P_{13}) = 0.00284{P_{13}}^2 + 8.6P_{13} + 126 + \lvert 100sin(0.084\times(40 - P_{13}))\rvert$	$55 \leq P_{13} \leq 120$
14	$C_{p14}(P_{14}) = 0.00284{P_{14}}^2 + 8.1P_{14} + 550 + \lvert 300sin(0.035\times(-P_{14}))\rvert$	$0 \leq P_{14} \leq 680$
15	$C_{p15}(P_{15}) = 0.00056{P_{15}}^2 + 8.1P_{15} + 309 + \lvert 200sin(0.042\times(-P_{15}))\rvert$	$0 \leq P_{15} \leq 360$
16	$C_{p16}(P_{16}) = 0.00056{P_{16}}^2 + 8.1P_{16} + 309 + \lvert 200sin(0.042\times(-P_{16}))\rvert$	$0 \leq P_{16} \leq 360$
17	$C_{p17}(P_{17}) = 0.00342{P_{17}}^2 + 7.7P_{17} + 240 + \lvert 150sin(0.063\times(60 - P_{17}))\rvert$	$0 \leq P_{17} \leq 180$
18	$C_{p18}(P_{18}) = 0.00342{P_{18}}^2 + 7.7P_{18} + 240 + \lvert 150sin(0.063\times(60 - P_{18}))\rvert$	$0 \leq P_{18} \leq 180$
19	$C_{p19}(P_{19}) = 0.00342{P_{19}}^2 + 7.7P_{19} + 240 + \lvert 150sin(0.063\times(60 - P_{19}))\rvert$	$0 \leq P_{19} \leq 180$
20	$C_{p20}(P_{20}) = 0.00342{P_{20}}^2 + 7.7P_{20} + 240 + \lvert 150sin(0.063\times(60 - P_{20}))\rvert$	$0 \leq P_{20} \leq 180$
21	$C_{p21}(P_{21}) = 0.00342{P_{21}}^2 + 7.7P_{21} + 240 + \lvert 150sin(0.063\times(60 - P_{21}))\rvert$	$0 \leq P_{21} \leq 180$
22	$C_{p22}(P_{22}) = 0.00342{P_{22}}^2 + 7.7P_{22} + 240 + \lvert 150sin(0.063\times(60 - P_{22}))\rvert$	$0 \leq P_{22} \leq 180$
23	$C_{p23}(P_{23}) = 0.00342{P_{23}}^2 + 7.7P_{23} + 240 + \lvert 150sin(0.063\times(60 - P_{23}))\rvert$	$40 \leq P_{23} \leq 120$
24	$C_{p22}(P_{24}) = 0.00342{P_{24}}^2 + 7.7P_{24} + 240 + \lvert 150sin(0.063\times(60 - P_{24}))\rvert$	$40 \leq P_{24} \leq 120$
25	$C_{p25}(P_{25}) = 0.00342{P_{25}}^2 + 7.7P_{25} + 240 + \lvert 150sin(0.063\times(60 - P_{25}))\rvert$	$55 \leq P_{25} \leq 120$
26	$C_{p26}(P_{26}) = 0.00342{P_{26}}^2 + 7.7P_{26} + 240 + \lvert 150sin(0.063\times(60 - P_{26}))\rvert$	$55 \leq P_{26} \leq 120$
27	$C_{c1}(O_1, H_1) = 0.034{O_1}^2 + 14.5O_1 + 2650 + 0.03{H_1}^2 + 4.2H_1 + 0.031O_1H_1$	CHP type A
28	$C_{c2}(O_2, H_2) = 0.0435{O_2}^2 + 36O_2 + 1250 + 0.027{H_2}^2 + 0.6H_2 + 0.11O_2H_2$	CHP type B
29	$C_{c3}(O_3, H_3) = 0.0345{O_3}^2 + 14.5O_3 + 2650 + 0.03{H_3}^2 + 4.2H_3 + 0.031O_3H_3$	CHP type A
30	$C_{c4}(O_4, H_4) = 0.0435{O_4}^2 + 36O_4 + 1250 + 0.027{H_4}^2 + 0.6H_4 + 0.011O_4H_4$	CHP type B
31	$C_{c5}(O_5, H_5) = 0.1035{O_5}^2 + 34.5O_5 + 2650 + 0.025{H_5}^2 + 2.203H_5 + 0.051O_5H_5$	CHP type C
32	$C_{c6}(O_6, H_6) = 0.072{O_6}^2 + 20O_6 + 1565 + 0.02{H_6}^2 + 2.34H_6 + 0.04O_6H_6$	CHP type D
33	$C_{c7}(O_7, H_7) = 0.0345{O_7}^2 + 14.5O_7 + 2650 + 0.03{H_7}^2 + 4.2H_7 + 0.031O_7H_7$	CHP type A
34	$C_{c8}(O_8, H_8) = 0.0435{O_8}^2 + 36O_8 + 1250 + 0.027{H_8}^2 + 0.6H_8 + 0.011O_8H_8$	CHP type B
35	$C_{c9}(O_9, H_9) = 0.0345{O_9}^2 + 14.5O_9 + 2650 + 0.03{H_9}^2 + 4.2H_9 + 0.031O_9H_9$	CHP type A
36	$C_{c10}(O_10, H_{10}) = 0.0435{O_{10}}^2 + 36O_{10} + 1250 + 0.027{H_{10}}^2 + 0.6H_{10} + 0.011O_{10}H_{10}$	CHP type B
37	$C_{c11}(O_{11}, H_{11}) = 0.1035{O_{11}}^2 + 34.5O_{11} + 2650 + 0.025{H_{11}}^2 + 2.203H_{11} + 0.051O_{11}H_{11}$	CHP type C
38	$C_{c12}(O_{12}, H_{12}) = 0.072{O_{12}}^2 + 20O_{12} + 1565 + 0.02{H_{12}}^2 + 2.34H_{12} + 0.04O_{12}H_{12}$	CHP type D

Unit No.	Cost Function	Capacity or FOR
39	$C_{h1}(T_1) = 0.038{T_1}^2 + 2.0109T_1 + 950$	$0 \leq T_1 \leq 2695.5$
40	$C_{h2}(T_2) = 0.038{T_2}^2 + 2.0109T_2 + 950$	$0 \leq T_2 \leq 60$
41	$C_{h3}(T_3) = 0.038{T_3}^2 + 2.0109T_3 + 950$	$0 \leq T_3 \leq 60$
42	$C_{h4}(T_4) = 0.052{T_4}^2 + 3.0651T_4 + 480$	$0 \leq T_4 \leq 120$
43	$C_{h5}(T_5) = 0.052{T_5}^2 + 3.0651T_5 + 480$	$0 \leq T_5 \leq 120$
44	$C_{h6}(T_6) = 0.038{T_6}^2 + 2.0109T_6 + 950$	$0 \leq T_6 \leq 2695.5$
45	$C_{h7}(T_7) = 0.038{T_7}^2 + 2.0109T_7 + 950$	$0 \leq T_7 \leq 60$
46	$C_{h8}(T_8) = 0.038{T_8}^2 + 2.0109T_8 + 950$	$0 \leq T_8 \leq 60$
47	$C_{h9}(T_9) = 0.052{T_9}^2 + 3.0651T_9 + 480$	$0 \leq T_9 \leq 120$
48	$C_{h10}(T_{10}) = 0.052{T_{10}}^2 + 3.0651T_{10} + 480$	$0 \leq T_{10} \leq 120$

Prohibited operating zones for POUs [20] ([MW, MW])

Unit No	Prohibited operating zone no.		
	1	2	3
1	[180, 200]	[260, 335]	[390, 420]
2	[30, 40]	[180, 220]	[305, 335]
3	[30, 40]	[180, 220]	[305, 335]
10	[45, 55]	[65, 75]	–
11	[45, 55]	[65, 75]	–
14	[180, 200]	[260, 335]	[390, 420]
15	[30, 40]	[180, 220]	[305, 335]
16	[30, 40]	[180, 220]	[305, 335]
23	[45, 55]	[65, 75]	–
24	[45, 55]	[65, 75]	–

heat and power economic dispatch problems by an improved genetic algorithm and a new constraint handling strategy," *Applied energy*, vol. 237, pp. 646–670, 2019.

39. A. Nazari and H. Abdi, "Solving the combined heat and power economic dispatch problem in different scale systems using the imperialist competitive harris hawks optimization algorithm," *Biomimetics*, vol. 8, no. 8, p. 587, 2023.

Matrix B (10-7/MW) [29]

92	26	48	99	12	56	53	54	28	90	26	21	99	76	39	37	62	44	84	50	74	39	68	20	86	89	74	62	36	51	98	04	99	51	74	66	36	66
21	93	27	87	82	83	48	02	79	14	64	70	30	73	71	29	02	37	08	12	53	92	39	14	34	07	16	51	41	8	80	82	84	47	20	05	37	15
64	97	31	29	95	68	07	51	18	50	56	14	12	99	82	21	70	18	24	86	23	49	75	34	60	09	76	15	60	66	53	27	39	38	12	83	75	40
62	15	15	83	10	56	10	97	93	29	41	63	34	84	46	78	19	75	57	45	39	57	37	30	42	17	90	72	50	77	87	78	33	61	08	84	82	89
81	12	76	66	70	77	27	97	92	59	07	89	80	72	22	60	85	22	23	52	20	78	20	55	79	88	99	77	02	68	33	14	09	13	75	39	50	11
73	02	53	69	29	69	86	40	61	05	32	73	38	54	65	73	73	68	04	97	80	73	32	25	40	97	03	55	85	90	78	38	98	23	74	95	37	90
65	45	59	71	50	80	43	01	04	04	97	19	21	92	21	13	80	65	99	98	08	20	19	22	50	70	49	23	85	07	55	41	63	01	10	88	32	74
47	89	28	10	05	43	38	05	62	78	24	17	25	27	65	48	98	36	98	80	12	58	65	78	24	11	53	76	25	16	48	72	16	64	85	34	12	90
56	07	98	21	82	95	62	51	42	35	06	29	19	11	06	83	83	11	76	53	47	41	71	22	21	54	98	13	16	20	43	13	51	30	69	68	86	78
98	22	53	70	74	70	92	24	49	74	98	15	82	66	97	47	27	85	58	37	55	20	79	98	06	10	38	95	57	91	57	23	37	14	25	50	28	99
05	33	21	60	14	97	42	22	14	94	03	15	56	36	87	74	05	60	32	65	32	79	92	11	27	44	69	77	75	95	67	97	13	76	75	10	25	35
39	05	52	96	22	27	50	96	46	36	98	02	28	21	08	62	36	39	66	61	54	18	33	71	64	10	03	59	01	24	67	39	98	74	23	03	11	67
55	18	77	45	89	88	77	38	29	48	58	42	23	05	82	00	96	17	91	29	13	44	48	17	53	76	71	51	93	52	43	20	83	11	90	79	47	69
42	80	52	77	88	51	24	62	02	11	82	05	87	68	51	94	05	09	07	43	36	12	89	83	20	52	76	35	67	38	52	96	85	98	72	45	67	34
15	89	52	34	01	17	13	93	83	31	79	62	74	37	21	06	46	44	73	64	16	52	36	85	99	70	36	60	92	30	58	61	51	36	85	56	99	15
86	82	11	8	96	35	29	47	07	55	98	26	65	59	05	37	38	50	32	47	53	92	68	54	41	33	49	76	14	38	37	29	57	10	88	81	73	15
43	28	05	84	53	10	90	42	17	09	26	45	22	38	02	08	81	13	67	57	84	35	99	14	32	28	44	88	26	92	40	84	76	86	85	36	03	16
27	79	37	18	32	22	48	06	02	78	95	46	99	12	46	72	07	54	35	95	93	30	91	01	64	81	45	28	16	27	35	22	10	46	86	93	11	38
23	23	61	17	79	62	51	62	48	28	93	32	57	72	53	20	94	93	88	66	81	02	91	40	83	18	62	77	66	89	33	53	69	05	69	78	93	26
82	45	99	46	15	18	73	02	57	93	45	13	02	74	47	11	31	11	76	09	33	25	91	99	10	52	86	18	83	59	33	73	72	88	61	10	28	38
30	10	77	12	31	86	21	81	85	16	20	09	72	11	89	16	44	15	67	51	55	46	15	46	44	92	62	88	06	72	40	94	85	33	73	81	84	15
42	91	51	73	47	01	37	96	45	92	31	05	55	74	73	87	57	78	02	11	62	31	69	73	92	15	53	75	59	67	25	11	73	16	32	69	57	38
62	39	37	06	12	52	53	38	56	04	10	89	12	41	68	69	29	51	85	38	89	42	12	76	13	65	64	06	36	45	01	05	66	90	05	42	18	10
90	63	27	47	64	58	06	35	48	51	54	65	80	08	10	73	21	45	31	41	74	26	65	20	78	84	38	74	06	96	60	71	61	30	31	78	04	27
88	90	95	36	71	37	05	98	92	80	86	73	19	89	63	28	25	67	47	56	82	08	10	97	44	63	97	13	95	77	53	01	38	74	41	89	77	16
33	94	10	17	23	23	58	59	85	25	09	72	04	01	63	02	75	83	53	61	33	15	91	14	66	60	57	37	54	93	96	87	21	97	91	82	28	08
58	47	12	37	05	66	67	63	60	43	26	73	55	91	45	10	27	39	99	38	68	50	37	35	75	81	90	06	98	19	64	91	48	34	89	25	97	88
71	50	19	43	68	28	32	42	87	42	06	28	61	39	89	12	14	48	88	84	64	97	52	17	48	58	54	66	73	21	70	93	76	68	60	88	20	48
61	37	00	56	55	58	15	99	27	71	16	31	56	30	29	90	28	37	65	46	83	49	85	55	01	80	12	91	94	19	34	98	46	23	46	11	88	38
86	92	13	75	61	81	05	41	80	26	81	25	82	19	26	33	31	35	05	44	96	96	71	81	90	84	09	46	88	23	20	60	74	82	25	96	72	82
25	08	32	40	64	33	13	03	94	36	14	91	45	86	60	76	77	74	12	02	01	21	83	95	40	27	80	06	91	69	35	60	82	14	67	05	73	32
47	06	78	01	66	16	64	98	55	45	60	51	30	59	10	76	51	43	02	34	51	29	95	20	30	74	45	13	20	66	86	13	59	68	73	94	70	74
23	13	92	16	91	38	32	56	99	96	71	05	56	10	90	11	37	72	87	31	15	91	89	84	27	56	77	72	41	44	51	35	34	35	13	61	09	22
50	78	73	39	77	31	21	48	89	41	86	91	81	02	55	37	59	05	85	28	48	23	65	22	07	36	94	24	13	05	46	47	33	23	63	67	03	52
15	18	18	33	85	64	80	30	41	22	46	26	15	41	81	63	72	93	42	59	15	66	82	12	08	11	07	54	54	46	58	97	44	27	63	30	36	26
88	50	49	14	07	43	05	77	08	24	02	29	15	44	85	61	80	65	94	04	69	72	64	95	92	34	13	22	63	51	37	79	32	90	50	90	42	41
07	97	52	85	62	91	48	14	23	56	49	10	11	38	51	65	95	79	59	31	67	73	62	42	63	48	47	89	08	41	04	65	42	66	72	41	87	48
15	77	36	28	73	37	27	83	02	53	22	77	12	90	39	71	84	72	50	35	58	25	68	44	53	98	50	24	60	97	12	66	97	85	02	03	56	13