Tri-level optimization-based resilient island city distribution network planning against terrorist attacks

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Power distribution networks are known as a key infrastructure of smart cities. The safe operation of such networks is generally threatened by terrorist agents to disrupt the activities of smart cities. Therefore, distribution network planners attempt to find a solution for increasing the resilience of a distribution system against terrorist attacks. The hardening of distribution lines and allocation of electrical energy storage units are generally performed for reaching planner goals. However, investment for obtaining a resilient distribution network against terrorist attacks is limited by a hardening budget. This budget can be increased by considering other distribution network plans (such as increasing the expansion of renewable energies) and using the geographical features of the studied distribution networks. This paper proposes a tri-level optimization problem for allocating and sizing the offshore pumped storage units for increasing an Island city distribution network resiliency against terrorist activities and minimizing the network operation cost by solving the stochastic nature of ocean wave generation powers. The proposed tri-level problem is solved based on the Benders decomposition method using primal cuts. The simulating results obtained by using the IEEE distribution test system validate the effectiveness of the proposed tri-level model and highlight that offshore pumped storage units can enhance the resiliency of a distribution system located on an island against terrorist attacks and reduce the total operation cost during normal conditions.

keywords: *Defender-attacker-defender model, offshore pumped storage, distribution network resiliency*

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1. INTRODUCTION

A. Motivation

In recent years, the impacts of distribution networks on smart cities' features have been demonstrated. Therefore, terrorist agents attempt to damage such networks for disrupting smart cities' activities. To respond to terrorist activities, the distribution lines' hardening projects are generally defined by the distribution network planner to improve distribution networks' resiliency against terrorist attacks. The execution of hardening projects needs heavy costs for reducing the impact of terrorist attacks on distribution networks. Therefore, the storage units' advantages such as their ability to face the stochastic nature of renewable generation [1], reducing the total operation cost [2], and decreasing the environmental pollution [3] have motivated the allocation of such units to improve the distribution network resiliency. A type of storage unit is an offshore pumped storage unit that can be installed in an Island city in order to achieve the overall advantages of storage and improve the resilience of the network against terrorist incidents.

B. Literature Review

As mentioned in the above description, the distribution line hardening projects, which include installing the undergrounding distribution lines and reinforcing of pole, are the most impressive method to improve distribution network resiliency against terrorist attacks. There are various studies which have determined an optimal hardening project by considering natural and terrorist threats. Such studies have commonly defined a model formulated based on the N-K contingency problem. Then, the defined problem has been solved by considering the type of model. Authors of reference [4] defined a bi-level problem to evaluate power system vulnerability under different contingency conditions. Also, a practical approach for detecting hardening strategies for increasing the resiliency of the system by using an iterative algorithm has been proposed in [5]. The mentioned algorithm selects the line with the highest load in each iteration. The bi-level hardening models are extended to a tri-level model known as the defender-attacker-defender model to find the best hardening strategy. A defended-attacked-defended model includes tree players. The first player is the distribution system planner, which selects the defended lines to harden them in order to minimize the terrorist damages measured by the load shedding value. The worst case of the attacked line is determined by the second player, which models terrorist behaviors. A terrorist agent selects a group of distribution lines for maximizing the load shedding value. In the tri-level models, terrorist behavior is modeled by a binary vector. The size of this vector is equal to the number of distribution lines. The third player is used to model distribution system behavior after a terrorist attack. The distribution network operation redispatches an attacked system to reduce the damages. In the first step of solving a defender-attacker-defender model, references generally convert it into an equivalent bi-level problem by using the dual theory. However, the binary vector used to model terrorist behavior is a constraint that is not allowed to convert an equivalent bi-level problem into a single problem. Therefore, such references adopt different optimization techniques to solve the defender-attacker-defender model. In [6], an implicit enumeration algorithm is applied to solve the model and find the protected lines. The enumeration algorithm is based on the branch and bound model, which assumes that an optimal hardening plan should include at least one critical line. This reference defined the critical lines as lines attacked without considering hardening projects. Also, the enumeration algorithm has been improved in a group of studies such as [7, 8]. Another technique used to solve the tri-level model is Benders decomposition [9, 10]. In some references, the equivalent bi-level problem has been reformulated as a master problem, which is used to model planner decision with a constant value for terrorist variables, and a sub-problem defined based on a lower-level problem and constant value for planner variables. Such variables and the upper and lower bounds of the problem are updated in each iteration. When the difference of upper and lower bounds of the problem is lower than a critical value, the algorithm is ended and the optimal hardening plan is obtained. The third method used to solve the equivalent bi-level problem is the column-and-constraint generation algorithm. Authors of reference [11] argue that the column-and-constraint generation algorithm is similar to solving the bi-level problem based on the Benders decomposing approach. This technique has been utilized to identify and harden the vulnerable ingredients of connected gas and electric networks from terrorist attacks in [12]. Also, reference [13] used this technique to deal with the power grid hardening problem by considering uncertain attacks and load types. Authors of reference [14] adopted this technique to solve the tri-level to select an optimal defense plan for a distribution network. Finally, some researchers employed the game theory and the Nash equilibrium point concept to reach the optimal solution of the defender-attacker-defender model [15, 16]. In [17], a problem of identifying the critical component of the water supply network is formulated based on the defender-attacker-defender model and is solved by using a Bayesian Stackelberg game model. In recent years, some researchers proposed various models to allocate distribution generation units and storage system units to enhance the resiliency of power systems. Authors of reference [18] presented a resilient distribution network planning problem for coordinating the hardening of distribution lines and the allocation of distributed generation units to minimize the system damage during a natural disaster. Moreover, reference [19] proposed a scheduling model for enhancing the resilience of a micro-grid including photovoltaic generation units and battery storage during extreme weather conditions. To improve the distribution system resiliency against terrorist attacks, authors of reference [20] proposed a model for sizing a storage unit for a micro-grid. This reference neglected the damage of distribution lines. Also, the model is not based on the defended-attacked-defended model. Therefore, it can be guaranteed that the worth attacked condition is considered. In [21], a system planning approach for enhancing the resiliency of connected gas and electrical systems is proposed by considering the thermal storage. Also, the attacker plan is modeled based on scenarios. Similar to the previous study, this reference did not consider the defender-attacker-defender model to take into account the worst case.

C. Contribution and Paper Structure

One can conclude that the coordinated distribution line hardening and energy storage system based on the defender-attacker-defender model worst case of terrorist attack damages is not considered. Also, the storage planning problems are strongly related to the type of storage units. Thus, in this paper, a tri-level island distribution network planning, including the installing of offshore pumped hydro-storage units, and hardening distribution lines model is proposed in order to minimize attack damages, reduce the total operation cost, and enhance the expansion of renewable energy such as ocean wave units during normal conditions. The remainder of this paper is organized as follows: Section 2 presents the mathematical formulation of the proposed model. The solution procedure is presented in Section 3. Section 4 describes case studies. Simulation results are also discussed in this section. Finally, conclusions are drawn in Section 5.

2. MATHEMATICAL RESILIENT BASED PLANNING MODEL

In this section, the mathematical planning model is provided for each player of the defended-attacked-defended model. As mentioned in the Introduction, a defended- attacker-defended model has three players. In the first description of this section, the mathematical model of the distribution network planner is formulated. In the next description, the terrorist attacker strategies are defined in the second level of the optimization problem. Finally, the distribution network operator model is provided in the third level of the optimization problem to reduce attack damages.

A. Distribution network planner model

In the proposed model, an Island distribution network planner should be given a defense strategy including distribution line hardening and allocating and sizing of off-shore pumped hydro storage units for minimizing terrorist damages during an attack condition, which is measured by the load shedding value, and reducing the total Island operation cost, which is obtained by storage charging and discharging, during normal conditions. Therefore, the objective function of distribution network planner model is classified in to three terms. This objective function is illustrated in (1).

$$\min_{d,P_i^F,P_i^G,\Delta P_i^D,\delta_i,\eta,u_n} \sum_{s=1}^{S} C_{storage} V_{resvior}^{s} + \\ (\sum_{t=1}^{T} \sum_{j=1}^{G} a_j P_{j,i,t}^{Gn^2} + b_j P_{j,i,t}^{Gn} + c_j) + T \times VOLL \times (\Delta P_n^{Dn} + \Delta Q_n^{Dn}) \tag{1}$$

In this objective function, the installing cost of storage unit is minimized. The off shore pumped hydro storage cost is a function of volume of storage reservoir ($V^{s}_{resvior}$). Also, $C_{storage}$ is used to model cos of storage unit construction and installing. The operation cost during normal conditions is minimized in the second term of objective function. The operation cost is obtained based on calculating cost of each thermal generation units, which is formulated by considering the generated power of each thermal unit determined by $P^{Gn}_{j,i,t}$. Also, the quadratic, linear and constant terms of the generation cost function of each unit are considered by a_j, b_j and c_j, respectively. In the third term of objective function, the terrorist attack damages is minimized during worst case of attacks. The terrorist damages are measured by the value of load shedding illustrated by ΔP^{Dn}_{n}, ΔQ^{Dn}_{n}. This factor is multiplied in the large value ($VOLL$) and planning horizon (T) for comparable the three terms of the objective function. Each term of objective function should be minimized as well as satisfied the related constraints. The constraint related to improve the resiliency of distribution network against terrorist attacks is hardening and allocating budget. The distribution network planer can be hardened a limited distribution lines and installed a limited off-shore pumped hydro storage units. This limitation is named by hardening constraint and modeled in (2).

$$\sum d_l \leq K_{line} \text{and} \sum u_{n'} = k_{storage} \tag{2}$$

In this constraint, two binary vectors are applied to detect the defended lines and buses, selected to install the offshore pumped hydro-storage units. The binary vector illustrated by d_l models the defended lines. When the l^{th} member of this vector is equal to one, the l^{th} ine is selected for hardening. The number of hardened lines should be lower than a certain value, which is related to the hardening budget and illustrated by K_{line}. The buses in which the storage unit is installed are determined by using the binary vector shown by $u_{n'}$. In this vector, n' is a subset of the distribution network bus set, which highlights the beach buses. When the n^{th} member of this vector is equal to one, the n^{th} bus is selected to install the offshore pumped hydro-storage unit. The number of buses, in which the storage units are installed, should be lower than a certain number, which is related to the expansion budget and is illustrated by $K_{storage}$.
In addition, the first term of the objective function should be minimized by considering the technical and economic constraints. The first constraint of the first term of the objective function is related to the distribution line power flow. In this paper, the DistFlow model is used to calculate the distribution line power flow. This model has been utilized in several studies such as [22, 23]. The nonlinear form of this model is formulated as:

$$\sum_{h|(n,h)\in L} p_{nh} = p_{mn} - r_{mn}\frac{p^2_{mn}+q^2_{mn}}{v^2_n} - P_n \tag{3}$$

$$\sum_{h|(n,h)\in L} q_{nh} = q_{mn} - x_{mn}\frac{p^2_{mn}+q^2_{mn}}{v^2_n} - Q_n \tag{4}$$

$$v^2_j = v^2_i - 2(r_{ij}p_{ij} + x_{ij}q_{ij}) + (r^2_{ij}+x^2_{ij})(\frac{p^2_{ij}+q^2_{ij}}{v^2_i}) \tag{5}$$

In this equations, the active and reactive power flow of line-connected buses n-h and m-n is shown by p_{nh} and p_{mn}, respectively. The impedance and reactance of such lines are modeled by using p_{ij}, x_{ij}. The active and reactive load power located at the n^{th} distribution bus is illustrated by P_n and Q_n, respectively. Also, the linear form of such equations is used to construct the power flow constraints of the proposed planning model. Such constraints are listed as follows [24, 25]:

$$\sum_{h|(n,h)\in L} p'^{t}_{nh} = p'^{t}_{mn} - P_n + P'^{g,t}_{n} + P'^{ow,t}_{n} + P^{s,t}_{'n,dis}u_{n'} - P'^{s,s}_{n,ch}u_{n'} \tag{6}$$

$$\sum_{h|(n,h)\in L} q'^{t}_{nh} = q'^{t}_{mn} - Q_n + q'^{g,t}_{n} \tag{7}$$

$$v_j = v_i - \frac{(r_{ij}p^{t}_{'ij} + x_{ij}q'^{t}_{ij})}{V_0} \tag{8}$$

$$P^{\min}_n \leq P'^{g,t}_n \leq P^{\max}_n \tag{9}$$

$$Q^{\min}_n \leq q'^{g,t}_n \leq Q^{\max}_n \tag{10}$$

where the bus voltage of the nth distribution bus is modeled by v_n. Other constraints of the second term of the objective function include the thermal capacity generation, offshore pumped hydro-storage unit constraints, and power balancing limitation. In these equations, the thermal generation and ocean wave powers located in the n^{th} bus are illustrated by $P'^{g,t}_n$ and $P'^{ow,t}_n$, respectively. Also, $P'^{s,t}_{n,dis}$ and $P'^{s,t}_{n,ch}$ are used to model the charging and discharging power of the pump-storage hydro-power, respectively. The power of the thermal unit should satisfy the minimum (P^{min}_g) and maximum (P^{max}_g) production level limitations. The limitation formulated in (9) ensures that this constraint is satisfied. Constraint (11) is applied to ensure that the bus voltage is lower than its maximum value and higher than its minimum value. The offshore hydro-pumped storage unit constraints are provided in (12)-(18). The discharging and charging power of the pumped-storage unit is computed in (12) and (**??**), respectively. In these equations, the turbine and pumped converter factors are denoted by σ^T_s and σ^P_s, in that order. Also, the water volume of the offshore hydro-storage reservoir is updated by using (14), where the pump and turbine of water flow are represented by $f'^{P,t}_s$ and $f'^{T,t}_s$, respectively. The water volume of the offshore hydro-pumped storage reservoir should be lower than the lower level values (0) and higher than the upper-level value ($V^s_{reservoir}$). Therefore, Constraint (15) is applied to model this limitation. Water extraction from the reservoir to generate power is limited by Constraint (16), whereas water extraction from the sea for pumping is limited by Constraint (17). In the planning, the volume of the offshore hydro-pumped storage reservoir should be lower than a maximum volume. The mentioned limitation is satisfied by (18). Finally, the scheduled power of the ocean wave generation unit should be lower than the forecasted value, which is modeled in (19).

$$-v^{\max}_n \leq v'^{t}_n \leq v^{\max}_n \tag{11}$$

$$P^{ps,t}_{'dis} = \sigma^T_s f'^{T,t}_s \tag{12}$$

$$P^{s,t}_{'ch} = \sigma^P_s f'^{P,t}_s \tag{13}$$

$$V'^{p,t} = V'^{p}(t-1) + q^{P,t}_{'p} - q^{T,t}_{p} \tag{14}$$

$$0 \leq V^{s,t} \leq V^{s}_{reservoir} \tag{15}$$

$$0 \leq f^{p,t}_{s} \leq F^{\max}_{p} \tag{16}$$

$$0 \leq f^{T,t}_{s} \leq F^{\max}_{p} \tag{17}$$

$$V^{s}_{reservoir} < V^{\max} \tag{18}$$

$$P'^{ow,t}_{n} < P'^{ow,t}_{n,forecast} \tag{19}$$

B. Distribution network terrorist model

In this subsection, the mathematical model of the terrorist agent is formulated based on maximizing the load shedding value. The objective function of this agent is written in (20).

$$\sum_{n\in N} \Delta P^{d*}_{n} = OF^{middle}_{1} = \max_{a} \sum_{n\in N} \Delta P^{d'}_{n} + \Delta Q^{d'}_{n} \tag{20}$$

Where a_l denotes a binary variable. If the i^{th} transmission line is selected for attack by the disruptive agent, this variable is equal to 0; otherwise, it is equal to 1 for non-attacked transmission lines. This equation models the disruptive attack plan. Consequently, it can be classified as the attack objective function of the disruptive agent. The disruptive agent should optimize the objective functions and satisfy the constraint illustrated in (23). This constraint is considered to ensure that the disruptive agent does not select a defender component.

$$a_l \geq d_l \tag{21}$$

C. Distribution network operation model

The system operator schedules the thermal generation units and pump-storage hydro-power to minimize the load shedding value during a disruptive attack. The system operator's objective function should be minimized by satisfying the technical system constraint. The mathematical model of this player is formulated as:

$$\sum_{n\in N} \Delta P^{d'}_{n} = OF^{lower}_{1} = \min_{P^{l},P^{g},P^{dis},\delta,\Delta P^{d}_{n}} \sum_{n\in N} \Delta P^{d}_{n} + \Delta Q^{d}_{n} \tag{22}$$

$$\begin{aligned} \sum_{h|(n,h)\in L} p_{nh} =& p_{mn} - P_{n} + P^{g}_{n} + P^{ow}_{n} + \\ & P^{s}_{n,dis} u_{n'} - P^{s}_{n,ch} u_{n'} + \Delta P^{d}_{n} \qquad (\lambda^{p}_{n}) \end{aligned} \tag{23}$$

$$\sum_{h|(n,h)\in L} q_{nh} = q_{mn} - Q_{n} + q^{g}_{n} + \Delta Q^{d}_{n} \quad (\lambda^{q}_{n}) \tag{24}$$

$$v_j = v_i - \frac{(r_{ij} p_{ij} + x_{ij} q_{ij})}{V_0} \quad (\nu_n) \tag{25}$$

$$P^{min}_{g} \leq P^{g}_{n} \leq P^{max}_{g} \quad (\underline{\gamma}^{p}_{g}, \bar{\gamma}^{p}_{g}) \tag{26}$$

$$Q^{min}_{g} \leq q^{g}_{n} \leq Q^{max}_{g} \quad (\underline{\gamma}^{q}_{g}, \bar{\gamma}^{q}_{g}) \tag{27}$$

$$v^{min}_{n} \leq v_{n} \leq v^{max}_{n} \quad (\underline{X}_{n}, \bar{X}_{n}) \tag{28}$$

$$P^{ps}_{dis} = \sigma^{T}_{s} f^{T}_{s} \quad (k^{T}_{s}) \tag{29}$$

$$P^{s}_{ch} = \sigma^{P}_{s} f^{P}_{s} \quad (k^{p}_{s}) \tag{30}$$

$$V^{s} = V^{s}_{initial} + f^{P}_{s} - f^{T}_{s} \quad (\omega_s) \tag{31}$$

$$V^{s} \leq V^{s}_{reservoir} \quad (\beta_s) \tag{32}$$

$$0 \leq f^{p}_{s} \leq F^{\max}_{s,p} \quad (\Omega_s) \tag{33}$$

$$0 \leq f^{T}_{s} \leq F^{\max}_{s,T} \quad (\theta_s) \tag{34}$$

$$P^{ow}_{n} < P^{ow}_{n,forecast} \quad (a^{ow}_{n}) \tag{35}$$

$$-P^{\max}_{l}(a_l) \leq p_{ij} \leq P^{\max}_{l}(a_l) \quad (\underline{\varphi}^{p}_{l,ij}, \bar{\varphi}^{p}_{l,ij}) \tag{36}$$

$$-Q^{\max}_{l}(a_l) \leq q_{ij} \leq Q^{\max}_{l}(a_l) \quad (\underline{\varphi}^{q}_{l,ij}, \bar{\varphi}^{q}_{l,ij}) \tag{37}$$

$$\Delta P^{d}_{n} \leq P_{n} \quad (a^{p}_{n}) \tag{38}$$

$$\Delta Q^{d}_{n} \leq Q_{n} \quad (a^{q}_{n}) \tag{39}$$

Constraints (36) and (37) are formulated to ensure that the attacked line is not operated during re-dispatching of the distribution network. Also, the description of other equations is similar to the description of constraints (6)-(19). In addition, Constraints (38) and (39) are considered to ensure that the load shedding value in each bus is lower than the load located on the mentioned bus.

3. SOLUTION PROCEDURE

In this paper, the tri-level planning optimization problem is converted into an equivalent bi-level optimization problem by applying the dual theory. However, due to the binary decision variable of the lower level of the equivalent problem, dual theory cannot be adopted to solve the equivalent bi-level problem. Therefore, the Benders decomposition method using primal cuts is derived to solve the equivalent bi-level optimal power system defense plan problem. In the first step of applying the Benders decomposition method using primal cuts, the master and sub-problems should be defined. In the master problem, the distribution system planner optimizes the objective function in each iteration based on the attacker plan obtained in the previous iteration. The mathematical model of the master problem is formulated by using constraints (1)-(19) and the following equations, which are added to the mentioned constraint to model the behavior of the distribution network operator and the terrorist agent. In these models, the behavior of the distribution network operator is considered by adding constraints written in (23)-(35), (38) and (39). Also, Constraints (43) and (44) are used to model the terrorist strategy obtained in the previous iteration. In such constraints, $a^{*}_{l,i}$ is a constant vector, and $d_{l,i}$ is a defense strategy for hardening lines. If a line is not hardened ($d_{l,i} = 0$) and selected by the terrorist agent ($a^{*}_{l,i} = 1$), the power flow of this line is set to zero. Therefore, optimal line hardening and storage allocation are obtained by solving the master problem based on the previous strategy of the distribution network operator and terrorist agent. Such optimal values are sent to the sub-problem to obtain the updated strategy of other model players.

$$\min_{d,P_i^F,P_i^G,\Delta P_i^D,\delta_i,\eta,u_n} \sum_{s=1}^{S} C_{storage}V^s_{resvior}+ \left(\sum_{t=1}^{T}\sum_{j=1}^{G} a_j P^{Gn^2}_{j,i,t} + b_j P^{Gn}_{j,i,t} + c_j\right)+T\times VOLL\times\eta \tag{40}$$

$$\eta \geq \sum \Delta P^D_{n,i} + \Delta Q^D_{n,i} \tag{41}$$

$$\sum d_l \leq K_{line}\text{and}\sum u_{n'} = k_{storage} \tag{42}$$

$$-(1-a^*_{l,i}+d_{l,i})P_l^{\max} \leq P^F_{l,ij} \leq (1-a^*_{l,i}+d_{l,i})P_l^{\max} \tag{43}$$

$$-(1-a^*_{l,i}+d_{l,i})Q_l^{\max} \leq Q^F_{l,ij} \leq (1-a^*_{l,i}+d_{l,i})Q_l^{\max} \tag{44}$$

After the definition of the master problem, the sub-problem should be defined to complete the proposed tri-level solution procedure. The sub-problem is formulated in (45)-(59).

$$\begin{aligned}&\max\sum_n(\lambda_n^p+a_n^p)P_n+\sum_n(\lambda_n^q+a_n^q)Q_n+\sum_n a_n^{ow}P^{ow}_{n,forcast}\\&+\sum_l P^{max}_{l,ij}(\bar{\varphi}^p_{l,ij}-\underline{\varphi}^p_{l,ij})a_l+\sum_l Q^{max}_{l,ij}(\bar{\varphi}^q_{l,ij}-\underline{\varphi}^q_{l,ij})a_l+\\&\sum_g\bar{\gamma}^p_g P_g^{max}-\sum_g\underline{\gamma}^p_g P_g^{min}+\sum_g\bar{\gamma}^q_g Q_g^{max}-\sum_g\underline{\gamma}^q_g Q_g^{min}\\&+\sum_n(v_n^{max}\bar{X}_n-v_n^{min}\underline{X}_n)+\sum_s(V^s_{initial}\omega_s+V^s_{reservoir}\beta_s\\&+F^{\max}_{s,p}\Omega_s+F^{\max}_{s,T}\theta_s)\end{aligned} \tag{45}$$

$$a_l \geq d_l^* \tag{46}$$

$$-\sum_{h|(n,h)\in L}(\lambda_n^p-\lambda_h^p)+\bar{\varphi}^p_{l,ij}+\underline{\varphi}^p_{l,ij}+\frac{r_{ij}}{v_o}\nu_n+\frac{x_{ij}}{v_o}\nu_n=0 \tag{47}$$

$$-\sum_{h|(n,h)\in L}(\lambda_n^q-\lambda_h^q)+\bar{\varphi}^q_{l,ij}+\underline{\varphi}^q_{l,ij} \tag{48}$$

$$\lambda_n^p+\bar{\gamma}_n^p+\underline{\gamma}_n^p \leq 0 \tag{49}$$

$$\lambda_n^q+\bar{\gamma}_n^q+\underline{\gamma}_n^q \leq 0 \tag{50}$$

$$\sum_{h|(i,j)\in L}(\nu_i-\nu_j)+\nu_n \leq 0 \tag{51}$$

$$\lambda_n^p+\bar{a}_n^p \leq 1 \tag{52}$$

$$\lambda_n^q+\bar{a}_n^q \leq 1 \tag{53}$$

$$\lambda_n^p+k_s^p \leq 0 \tag{54}$$

$$-\lambda_n^p+k_s^T \leq 0 \tag{55}$$

$$\lambda_n^p+\bar{a}_n^{ow} \leq 0 \tag{56}$$

$$k_s^p+\omega_s+\Omega_s \leq 0 \tag{57}$$

$$k_s^T+\omega_s+\theta_s \leq 0 \tag{58}$$

$$\omega_s+\beta_s \leq 0 \tag{59}$$

According to the sub-problem formulation, the existence of a non-linear part of the objective function increases the nonlinear nature of the problem by multiplying a binary variable by the continuous variables shown in $P^{max}_{l,ij}(\bar{\varphi}^p_{l,ij}-\underline{\varphi}^p_{l,ij})a_l$. In this paper, to enhance the simulation speed and achieve the best possible solution, this part of the problem has been linearized using two auxiliary variables and by adding additional constraints to the model. For this purpose, the auxiliary variable t has replaced the nonlinear part of the objective function $P^{max}_{l,ij}(\bar{\varphi}^p_{l,ij}-\underline{\varphi}^p_{l,ij})a_l$. By replacing this auxiliary variable, the objective function of the problem is written as the equation shown in (60).

$$\begin{aligned}&\max\sum_{i,j,s}t_l+\sum_{i,s}(\bar{s}_i^s+\lambda_i^s)P_i^d+\sum_{i,s}\bar{\gamma}_i^s P_i^{g,max}-\sum_{i,s}\underline{\gamma}_i^s P_i^{g,max}+\\&\sum_s(V_u^{p,s}(t-1)\kappa_i^s+V_L^{p,s}(t-1)v_i^s+Q_p^{\max}\beta_i^s-\sigma_p^T\Omega_i^s+\\&\bar{\omega}_i^s P_U^{p,max}-\underline{\omega}_i^s P_L^{p,max}+\underline{\omega}_i^s P_L^{p,max}-\underline{\theta}_i^s P_L^{p,min}\end{aligned} \tag{60}$$

The value of the replaced auxiliary variable is determined by the following constraints that must be added to the simulation problem:

$$t_l=\left(\left(\bar{\phi}^s_{ij}-\underline{\phi}^s_{ij}\right)P^{\max}_{l,ij}\right)-h_l \tag{61}$$

$$\left(\bar{\phi}^s_{ij}-\underline{\phi}^s_{ij}\right)^{\text{min-value}}d_l \leq t_l \leq \left(\bar{\phi}^s_{ij}-\underline{\phi}^s_{ij}\right)^{\text{up-value}}d_l \tag{62}$$

$$\left(\bar{\phi}^s_{ij}-\underline{\phi}^s_{ij}\right)^{\text{min-value}}(1-d_l) \leq h_l \leq \left(\bar{\phi}^s_{ij}-\underline{\phi}^s_{ij}\right)^{\text{up-value}}(1-d_l) \tag{63}$$

By adding the above equations, the nonlinear nature of the sub-problem is reduced. Also, the model presented in the above equations must be solved by considering the constant values of the master problem results.

After forming the sub-problem, the proposed model is solved according to the algorithm shown in Algorithm 1. Both the main and the sub-problems should have information about the initial performance. In addition, the maser problem includes high-level variables that specify the buses located at the storage units and the hardening lines, while the sub-problem includes mid-level variables indicating the terrorist attack policy. The Benders decomposition algorithm requires that the low-level problem include high-level and medium-level variables. For this reason, the constraints shown in (43), (44) and (46) require a combination of high- and low-level decisions. The maser and sub-problems form lower and upper bounds, respectively, based on the optimal value of the objective function of sub problem and optimal value of η. In the first step of this algorithm, the lower (LB) and the upper (UB) bounds are set to infinitely negative and positive, respectively, and the repetition counter is set to zero. After making these settings, the sub-problem will be solved without considering the defense policy. After solving this problem, Z lines, which have the most vulnerability, are determined. After this stage, the attacker's initial policy is available to solve the master problem. In the second step of the algorithm, the master problem is solved according to the initial data, and the appropriate defense policy of the planner is calculated in this step. Defender policy data are transferred into the sub-problem to solve this problem. In the third step of the algorithm, the sub-problem is solved and the results, which include the attacker's attack policy, are transferred to the master problem. After performing this step, the lower limit will be updated according to the optimal value of η obtained by solving the master problem. Furthermore, if the value of the objective function of the sub-problem is less than the value of the upper bound, the value of this bound will also be updated. This algorithm continues, if the difference between the upper and lower bound is not less than the specified value, or the number of iterations does not reach its maximum value. A summary of the Benders decomposition algorithm is presented in Algorithm 1.

Algorithm 1. Benders decomposition algorithm

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 1: Determining the initial values of the upper and lower bounds
 2: Solve the sub-problem without considering the defense policy
 3: Determine the terrorist initial policy
 4: for i = 1: itermax do
 5:     Solve the master problem based on the terrorist initial policy
 6:     Send this iteration defense policy
 7:     Solve the sub-problem by constantly considering high-level
        variables (defense policy and storage location)
 8:     Update the lower bound value based on the master problem
        objective
 9:     if the sub-problem objective function < upper bound then
10:         Update the upper limit of the problem according to the value
            obtained from the upper limit objective function
11:     if the difference between the upper and lower bounds<$\varepsilon$ then
12:         Go to 11
13:     Go to 4
14: End of algorithm
```

4. NUMERICAL RESULTS

In this section, the efficiency of the proposed resilience-based distribution network planning is investigated. Two case studies are used to highlight the impact of hardening projects and storage unit allocation and sizing on reducing the terrorist damages and total operation cost during a terrorist attack and normal operation conditions, respectively. The proposed model is applied on the modified IEEE 33-node test system to model an Island distribution network.

A. Case study 1

The proposed resilience-based distribution network planning model is solved for a modified 33-node distribution network isolated from the network to simulate an island distribution network. Three thermal distribution generation units are inserted into the IEEE 33-node distribution network so that it can respond to the network load. Also, an ocean wave power plant with a capacity of 1.2 MW is considered in the modified test system. The basic load of the studied distribution network is plotted in Fig. 1 per bus. Table 1 presents information on the distributed generation power plants. Such power plants are installed in buses 13, 21, and 31, respectively.

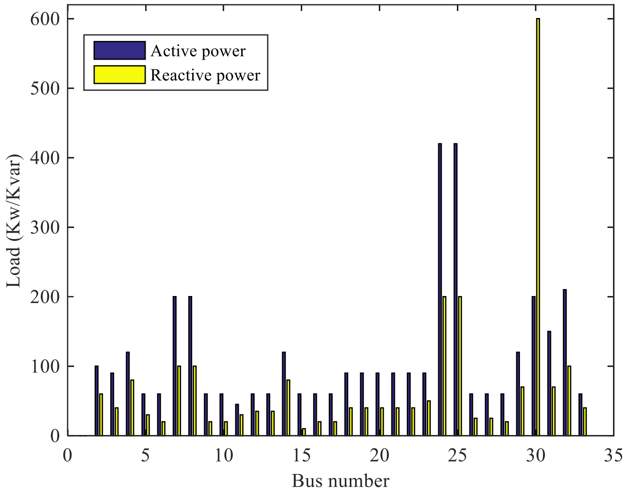

Fig. 1. The modified IEEE 33-node basic loading data.

Table 1. Modified IEEE 33-node generation unit data

Unit	Cost function			Pmax (MW)	Pmin (MW)	Installed bus
	a	b	c			
	$\$/MW^2$	\$/MW	\$			
G1	0.00028	8.1	55	2.2	0.3	13
G2	0.00324	7.74	24	1.5	0	21
G3	0.00284	8.6	12	2.2	0	31

In the first investigation of this case study, the studied distribution network's vulnerability against terrorist incidents is examined. According to the previous section, to obtain network vulnerabilities, it is sufficient to solve the model presented in the sub-problem by setting zero values to each of the high-level problem variables. The vulnerability of the distribution network is a function of the number of lines selected by the terrorist agent to deactivate them. Fig. 2 displays the change curve of the amount of load shedding values imposed on the network if the appropriate defense plan is not considered. Based on this figure, the rate of load shedding of the distribution network during the getting out of service of only one distribution line by the terrorist agent is equal to 930 kW, while this amount of load shedding is equal to 3325 kW with the getting out of service of nine distribution lines. As a result, it seems necessary to use a suitable defense plan to reduce this amount. The value of load shedding is increased by raising the number of attacked lines. In addition, the terrorist strategy for attacks line, obtained by solving the sub-problem without considering defense plans, is reported in Table 2 for the selected attacked line from 1 to 3 distribution lines.

Table 2. Worst case for attacking obtained by solving the sub-problem without considering defense plans

Number of attacked lines	Selected lines	Damage value (Kw)
1	12	930
2	12-21	1320
3	12-21-32	1820

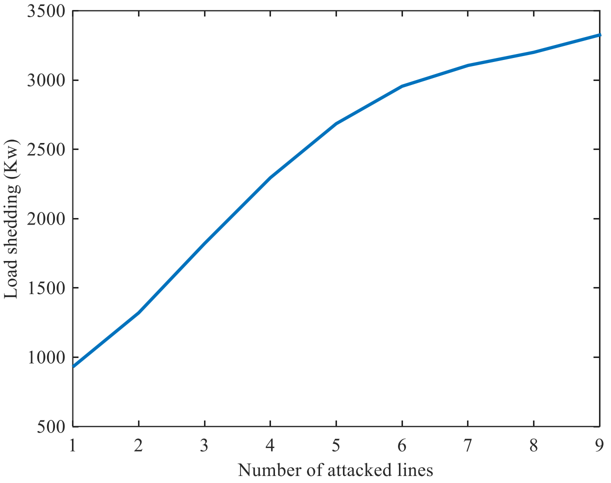

Fig. 2. Distribution network's vulnerability against terrorist incidents.

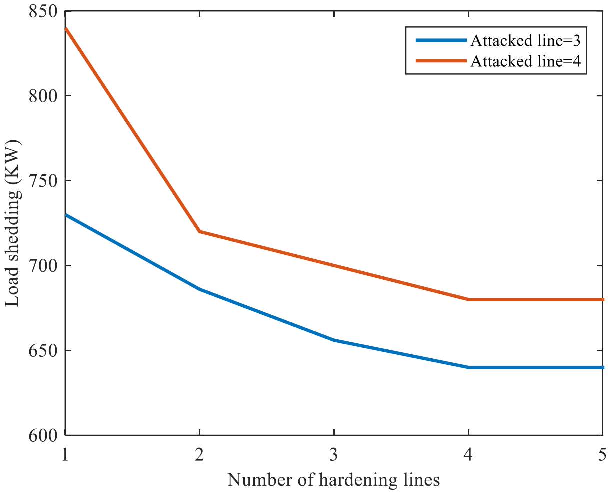

Fig. 3. Effect of line hardening on the rate of network load shedding against attacks.

In the second investigation of this case study, the effect of hardening of distribution lines against terrorist incidents has been studied. To this end, it is investigated whether increasing the distributed defenses lines obtained by implementing Algorithm 1 changes the amount of load shedding imposed on the network due to terrorist acts. Therefore, the rate of network load shedding for a fixed number of attacked lines and the increase in the number of lines that the network planner hardened are examined in this investigation. Fig. 3 illustrates the amount of load shedding imposed on the network in the case where several lines are hardened, in exchange for attacking three and four lines. According to this figure, the distribution lines hardening project can decrease the value of network load shedding by a certain amount. The distribution network is more vulnerable to terrorist incidents than the power transmission network since the transmission network can reduce load shedding depending on the amount of protected lines. However, in the distribution network, by increasing the protected lines, the amount of load shedding does not decrease below a certain amount and requires the use of distributed generation sources or storage units installed in a suitable place to reduce load shedding.

In the third investigation of this case study, the impact of the offshore pumped hydro-storage unit on reducing the rate of load shedding value has been evaluated. According to the problem formulation presented in Section 2, the offshore pumped hydro-storage unit location is limited by installation buss constraint (Only the beach buses are considered for the installation of this type of storage, which is equivalent to four buses in the modified IEEE 33-node distribution network) and can reduce the network load shedding. The results of this investigation have been obtained for the conditions in which the terrorist agent wants to damage the 2-7 distribution lines, and the situation has been investigated for the other cases as well. The results of sizing and allocating of this type of storage is reported in Table 3 by considering the pump and turbine efficiency as 0.89 and 0.82, respectively. According to this table, the storage capacity is equal to 1.8 MW. Fig. 4 reports the network load shedding rate for the installation of this storage unit and considering two hardening lines. The water volume of this reservoir is considered as 60% total capacity. Moreover, the total operation cost for one and two years scheduling horizon is depicted in Fig. 5. According to this figure, it can be concluded that this storage can reduce total cost about $7162869 and $19315048, respectively. The hydro-storage unit can store the spillage ocean wave power during normal conditions. The stored energy is used to decrease thermal power generation to respond to peak distribution load. Therefore, the total operation cost reduction is validated by considering the charging and discharging conditions of storage units.

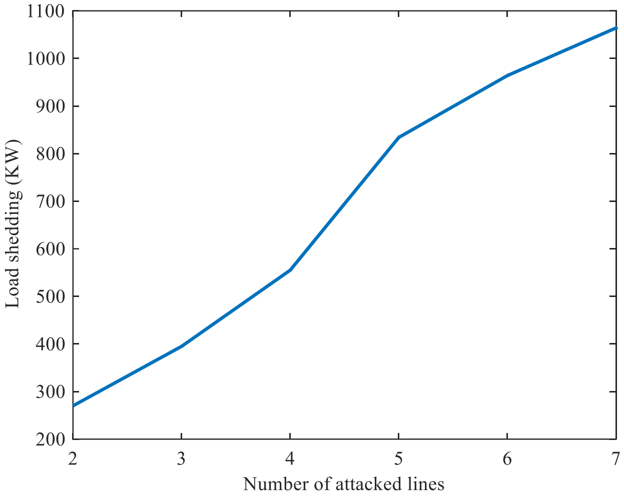

Fig. 4. Rate of load shedding imposed by the terrorist agent while taking into account the storage unit.

Table 3. Results of sizing and allocating off shore pumped storage

Parameter	Value
Installed bus	3
Reservoir volume	1200 m^3
Electrical storage capacity	1.8 MW

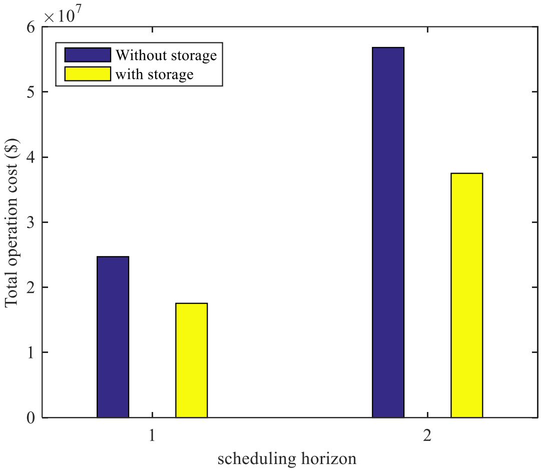

Fig. 5. Total operation cost with and without storage unit.

5. CONCLUSION

A tri-level optimization problem was solved for allocating and sizing the offshore pumped storage units and selecting the optimal hardening of the distribution line to increase an Island distribution network resiliency against terrorist activities and minimizing the network operation cost by solving the stochastic nature of ocean wave generation powers. The Benders decomposition method using primal cuts was applied to obtain optimal solutions of the proposed tri-level resilience-based distribution planning model. The simulating results obtained by using the IEEE distribution test system showed that a defense plan including an offshore pumped storage unit installing and two hardened distribution lines can improve distribution resiliency against terrorist attacks more than hardening eight distribution lines. In addition, such units can reduce ocean wave electrical production spillage. Also, simulation results indicated that the application of the offshore pumped hydro-storage can reduce about 30% of the total operation cost during normal distribution network operating conditions.

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